Math 227 Assignment 4

Due Thursday, March 8

1) For the following linear transformations, find the matrix of the transformation. For part b), also check that $P \cdot P = P$.

a) (1 point) $S : \mathbb{R} \to \mathbb{R}^4$, $S(t) = (5t, -6t, 3t/2, 0)$.

b) (3 points) $P : \mathbb{R}^2 \to \mathbb{R}^2$, $P(x, y) = ((x + y)/2, (x + y)/2)$.

c) (4 points) $T : \mathbb{R}^3 \to \mathbb{R}^2$, $T(x, y, z) = (x - 5y + 34z, 2x + 3z - 8y)$.

2) a) (2 points) Let $A \in M_2(\mathbb{R})$. Show that if $A = aI_2$, then $AB = BA$ for all $B \in M_2(\mathbb{R})$.

b) (4 points) For all $A \in M_2(\mathbb{R})$ where A is NOT a scalar multiple of I_2, find $C \in M_2(\mathbb{R})$ with $AC \neq CA$ (*Hint:* use matrices with a single nonzero entry.)

3) Find the inverse of the following matrices, then check that your answer is correct. Please don't find any inverses by hand, but do tell me how you obtained them, whether by a formula or Wolfram Alpha.

a) (2 points) $A = \begin{bmatrix} -1 & 2 \\ 4 & 6 \end{bmatrix}$.

b) (2 points) $B = \begin{bmatrix} 6 & 8 & 4 \\ 5 & 1 & -10 \\ 11 & 9 & 9 \end{bmatrix}$.

4) Find a single 3×3 matrix that, in homogeneous coordinates,

a) (1 point) rotates a 2-vector $\pi/6$ radians,

b) (2 points) shifts a 2-vector down by 3 and right by 12,

c) (1 points) scales a 2-vector up by 9, and finally

d) (3 points) does parts a)-c) in order, starting with a).