Math 227 Assignment 5

Due Thursday, March 5

1) a) (2 points) Write down the standard matrix for the reflection in $\mathbb{R}^2$ about the line $y = 5x$.

b) (3 points) If, for $m \neq 0$,

$$R = \frac{1}{1 + m^2} \begin{bmatrix} m^2 - 1 & 2m \\ 2m & 1 - m^2 \end{bmatrix},$$

show that $R^2 = I_2$.

2) Show that the following collections of vectors are subspaces of the indicated vector space.

a) (4 points) $\left\{ \begin{bmatrix} a & b \\ b & c \end{bmatrix} \;\middle|\; a, b, c \in \mathbb{R} \right\} \subseteq M_2(\mathbb{R})$,

b) (4 points) $\{(x, y, z, w) \mid 14x - 9y - 20z + w = 0\} \subseteq \mathbb{R}^4$

c) (4 points) $\{(a_n)_{n=1}^{\infty} \mid \lim_{n \to \infty} a_n = 0\} \subseteq S$.

3) (3 points) Let $S = \{(x, y) \; : \; |x| = |y|\} \subset \mathbb{R}^2$. Show that S is NOT a subspace of $\mathbb{R}^2$.

4) a) (2 points) If

$$A = \begin{bmatrix} 1 & 4 & -1 & 5 \\ 2 & 8 & -2 & 10 \\ 16 & 7 & 4 & 20 \\ 17 & 11 & 3 & 25 \end{bmatrix},$$

find two nonzero vectors in $\text{Null}(A)$ that are not scalar multiples of eachother.

b) (3 points) If $T : M_2(\mathbb{R}) \to M_2(\mathbb{R})$, $T(A) = A - A^t$, find the range and kernel of T.