Math 227 Assignment 6

Due Thursday, March 26

1) (5 points) If V is an n-dimensional vector space with basis $\{b_i\}_{i=1}^n$, we constructed the linear map $T : V \to \mathbb{R}^n$ given by

$$T\left(\sum_{i=1}^{n} a_i b_i\right) = \sum_{i=1}^{n} a_i e_i$$

where a_i is a scalar for all $1 \leq i \leq n$ and $\{e_i\}_{i=1}^n$ is the standard basis for $\mathbb{R}^n$. We claimed that T is an isomorphism. Check that T is both one-to-one and onto.

2) a) (2 points) Let

$$A = \begin{bmatrix} -5 & 7 \\ 2 & -6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & 6 \\ -4 & 12 \end{bmatrix}.$$

Compute $\det(A)$, $\det(B)$, and $\det(A+B)$. Is it true that $\det(A) + \det(B) = \det(A+B)$?

b) (3 points) Let $C \in M_n(\mathbb{R})$ and suppose C is invertible. Show that $\det(C^{-1}) = 1/\det(C)$. Do NOT use the definition, just use properties of the determinant and inverse!

3) For each of the following matrices, find all its eigenvalues, provide an associated eigenvector whose 2-norm is equal to 5, and check that $Ax = \lambda x$ for each matrix A, eigenvalue λ, and associated eigenvector x you found. For part a) do the check BY HAND.

a) (5 points) $\begin{bmatrix} 1 & 4 \\ -2 & 8 \end{bmatrix}$

b) (6 points) $\begin{bmatrix} -4 & 0 & -4 \\ 3 & 2 & 5 \\ -1 & 4 & 3 \end{bmatrix}$

4) (4 points) Let $\mathcal{S}$ is the space of sequences of real numbers and $T : \mathcal{S} \to \mathcal{S}$,

$$T((a_n)_{n=1}^{\infty}) = (0, a_1, a_2, a_3, \ldots).$$

Show that T has no eigenvalues.