Math 227 Assignment 6

Due Thursday, March 29

1) Find a basis $\mathcal{B}$ for each of the following subspaces, then construct an orthonormal basis $\mathcal{ONB}$ (you may use Wolfram Alpha). Check that each vector in $\mathcal{B}$ is in span($\mathcal{ONB}$).

a) (3 points) $W_1 = \text{ran}(A) \subseteq \mathbb{R}^3$ where $A = \begin{bmatrix} -1 & 6 & 5 \\ 2 & -12 & 0 \\ 0 & 0 & 10 \end{bmatrix}$,

b) (4 points) $W_2 = \{(x, y, z, w) \mid 14x - 9y - 20z + w = 0\} \subseteq \mathbb{R}^4$

2) Define $P : \mathbb{R}^2 \to \mathbb{R}^2$, $P\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ 3x \end{bmatrix}$.

a) (2 points) Find a matrix that implements P.

b) (3 points) Show that $P^2 = P$, but P is not an orthogonal projection.

3) (5 points) Check that $\{1, x, 2x^2 - 1, 4x^3 - 3x\}$ is a basis for $\mathbb{P}_3[x]$.

4) Let

$$W = \{(a_n)_{n=1}^{\infty} \mid \text{there is a } k \text{ with } a_n = 0 \text{ for } n \geq k\} \subset \mathcal{S}$$

where $\mathcal{S}$ is the vector space of all sequences of real numbers. In words, W is the space of all sequences that are eventually constantly zero.

a) (2 points) Write down a nonzero sequence in W and a sequence not in W.

b) (4 points) Find a basis for W.

c) (2 points) Is your answer from b) a basis for $\mathcal{S}$? Why or why not?