Math 227 Assignment 7

Due Thursday, April 5

1) For each of the following matrices, find all its eigenvalues, provide an associated eigenvector and check that $Ax = \lambda x$ for each matrix A, eigenvalue λ, and associated eigenvector x you found. For part a) do the check BY HAND.

a) (4 points) $\begin{bmatrix} 1 & 4 \\ -2 & 8 \end{bmatrix}$

b) (5 points) $\begin{bmatrix} -4 & 0 & -4 \\ 3 & 2 & 5 \\ -1 & 4 & 3 \end{bmatrix}$

2) (4 points) Let $\mathcal{S}$ be the space of sequences of real numbers and $T : \mathcal{S} \to \mathcal{S}$,

$$T((a_n)_{n=1}^{\infty}) = (0, a_1, a_2, a_3, \dots).$$

Show that T has no eigenvalues.

3) a) (3 points) Given the simplified link diagram between webpages P_1, P_2, and P_3 described by

- P_1 links to P_2 and P_3
- P_2 links to P_3 and P_1
- P_3 links to P_2,

find the PageRank of P_3, using $d = .85$ by i) constructing the link matrix A, then ii) finding the normalized matrix B, iii) calculating the PageRank matrix C, and finally iv) finding the associated eigenvector v with all positive entries whose 1-norm is equal to one and reading off the PageRank. NOTE: last I checked, Wolfram Alpha is VERY stupid here and cannot see that 1 is actually an eigenvalue.

b) (4 points) Same problem as a), except now P_1, P_2, and P_3 are related as follows:

- P_1 links to P_2

- P_2 links to P_1
- P_3 doesn't link to anything.

Observe how, with a small number of webpages, the PageRank can be skewed by an isolated site.

c) (5 points) Same problem as a), except now you have 5 pages P_1, P_2, P_3, P_4, and P_5, the pages are related by

- P_1 links to P_2
- P_2 links to P_3
- P_3 links to P_4 and P_5
- P_4 doesn't link to anything
- P_5 links to P_1, P_2, and P_4,

and you should find the PageRank of P_5.