Name:

Math 227 Exam 1

October 3, 2018

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.

1) a) (3 points) How many solutions can you have for a system of linear equations? Be sure to cover all possibilities.

b) (20 points) Solve the following system of equations BY HAND, using any manner at your disposal, or show there is no solution. SHOW YOUR WORK.

$$2x - z = 7$$
$$-6x + 10y + 5z = 7$$
$$9x + 25y - 10z = 7.$$

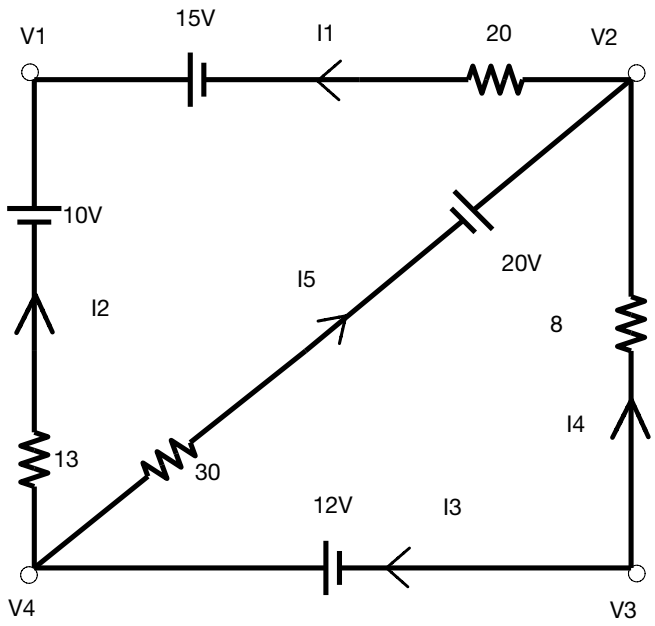
2) Find a quadratic interpolating polynomial through the points $(4, -2)$, $(6, 1)$, and $(9, 11)$ by

a) (12 points) writing down a system of linear equations that determines the coefficients of the polynomial, then

b) (6 points) producing an augmented matrix for the system, and finally

c) (6 points) row-reducing your matrix from b) and writing down the interpolating polynomial.

3) Consider the following electrical circuit (resistance is in Ohms):



a) (8 points) Find the edge-node incidence matrix A.

b) (6 points) Determine the resistance matrix R.

c) (8 points) Set up a matrix equation for finding the currents I_1, I_2, I_3, I_4, and I_5 and the potential differences between v_1, v_2, v_3, and v_4.

d) See the next page!

3) d) (continued) (9 points) If the row-reduced matrix you obtain is below, find the currents and potential differences.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5/747 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5/747 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 211/249 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -211/249 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 628/747 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 77405/747 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & -(1300/249) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -12 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

4) a) (10 points) If $v = \begin{bmatrix} 5 \\ -12 \end{bmatrix}$, find a unit vector whose angle with v is $30°$.

b) (12 points) If $w = \begin{bmatrix} 5 \\ -12 \\ 1 \end{bmatrix}$, show that there are infinitely many unit vectors whose angle with w is $30°$.

BONUS: (10 points) Suppose A is an $m \times n$ matrix, b is a vector in $\mathbb{R}^n$, and $Ab = 0$. Does this mean $b = 0$? Justify your answer.