Name:

Math 227 Exam 1

October 1, 2019

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.

1) a) (3 points) How many solutions can you have for a system of linear equations? Be sure to cover all possibilities.

b) (18 points) Solve the following system of equations BY HAND, using any manner at your disposal, or show there is no solution. SHOW YOUR WORK.

$$x - 2y + 5z = -1$$

$$3x + 6y - 8z = 23$$

$$9x + 6y - 12z = 21$$

2) Find a cubic interpolating polynomial through the points $(1, 6)$, $(-2, 5)$, $(3, -8)$ and $(5, 9)$ by

a) (10 points) writing down a system of linear equations that determines the coefficients of the polynomial, then

b) (10 points) producing an augmented matrix for the system, and finally

c) (6 points) row-reducing your matrix from b) and writing down the interpolating polynomial.

3) a) (6 points) For each of the following matrices in row-reduced echelon form, determine whether the associated system of linear equations has zero, one, or infinitely many solutions.

(i) $\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -119 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 0 & 1 & -3 \\ 0 & 1 & -7 & 81 \end{bmatrix}$

b) (15 points) For vectors

$$v = \begin{bmatrix} -1 \\ 7 \\ -3 \end{bmatrix}, \; w = \begin{bmatrix} -6 \\ 4 \\ -2 \end{bmatrix}, \text{ and } u = \begin{bmatrix} -36 \\ 5 \\ -4 \end{bmatrix},$$

find numbers x and y with $xv + yw + u = \vec{0}$.

4) Let $v = \begin{bmatrix} -15 \\ 8 \end{bmatrix}$.

a) (3 points) Compute the magnitude of v.

b) (12 points) Find a vector of length 3 whose angle with v is $45°$.

5) For ALL A arbitrary 2×2 matrices: let v be a 2×1 column vector.

(i) (5 points) Which of the following multiplications makes sense, Av or vA? Explain.

(ii) (12 points) Suppose in addition that the second row of A is a multiple of the first row. Show there is a nonzero vector v with $Av = \vec{0}$.