

$2R3$
 $+R2$

$$\begin{bmatrix} 1 & -2 & 5 & -1 \\ 0 & 12 & 0 & 72 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$R2/12$

$$\begin{bmatrix} 1 & -2 & 5 & -1 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{aligned} x &= 1 \\ y &= 6 \\ z &= 2 \end{aligned}$$

$2R2$
 $+R1$

$$\begin{bmatrix} 1 & 0 & 5 & 11 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$-5R3$
 $+R1$

$$\begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

-or-

$$\sum (x - 2y + 5z = -1)$$

$$3x - 6y + 15z = -3$$

$$- (3x + 6y - 8z = 23)$$

$$-12y + 23z = -26$$

$$-3 (3x + 6y - 8z = 23)$$

$$9x + 6y - 12z = 21$$

$$+ (-9x - 18y + 24z = -69)$$

$$-12y + 12z = -48$$

$$-y + z = -4$$

$$y = z + 4$$

+1

$$-12(z+4) + 23z = -26$$

$$-12z - 48 + 23z = -26$$

$$11z = 22$$

$$z = 2$$

$$y = z + 4 = 6$$

$$x - 2(6) + 5(2) = -1$$

$$x = -(+12 - 10) = 1$$

$$2) a) \quad y = ax^3 + bx^2 + cx + d$$

$$6 = a + b + c + d$$

$$5 = -8a + 4b - 2c + d$$

$$-8 = 27a + 9b + 3c + d$$

$$9 = 125a + 25b + 5c + d$$

b)

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 6 \\ -8 & 4 & -2 & 1 & 5 \\ 27 & 9 & 3 & 1 & -8 \\ 125 & 25 & 5 & 1 & 9 \end{bmatrix}$$

$$c) \begin{bmatrix} 1 & 0 & 0 & 0 & 641/840 \\ 0 & 1 & 0 & 0 & -419/140 \\ 0 & 0 & 1 & 0 & -4157/840 \\ 0 & 0 & 0 & 1 & 369/28 \end{bmatrix}$$

$$y = \frac{641}{840} x^3 - \frac{419}{140} x^2 - \frac{4157}{840} x + \frac{369}{28}$$

3) a) (i) one

(ii) none

(iii) infinitely many

$$b) \quad x \begin{bmatrix} -1 \\ 7 \\ -3 \end{bmatrix} + y \begin{bmatrix} -6 \\ 4 \\ -2 \end{bmatrix} + \begin{bmatrix} -36 \\ 5 \\ -4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-x - 6y = 36$$

$$7x + 4y = -5$$

$$-3x - 2y = 4$$

$$\text{ref} \begin{bmatrix} -1 & -6 & 36 \\ 7 & 4 & -5 \\ -3 & -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -13/2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x = 3, \\ y = -13/2$$

$$4) \quad a) \quad \|v\|_2 = \sqrt{15^2 + 8^2}$$

$$= \sqrt{225 + 64}$$

$$= \sqrt{289} = 17$$

$$b) \quad w = \begin{bmatrix} r \cos \theta \\ r \sin \theta \end{bmatrix} \quad \text{with } r=3$$

$$\text{angle of } v = \arctan\left(\frac{8}{-15}\right) + \pi$$

$$w = \begin{bmatrix} 3 \cos\left(\arctan\left(\frac{8}{-15}\right) + 45^\circ + 180^\circ\right) \\ 3 \sin\left(\arctan\left(\frac{8}{-15}\right) + 45^\circ + 180^\circ\right) \end{bmatrix}$$

$$\approx \begin{bmatrix} -2.87700 \\ -.8735 \end{bmatrix}, \begin{bmatrix} -.8735 \\ 2.87700 \end{bmatrix}$$

- or -

$$\omega = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\sqrt{x^2 + y^2} = 3, \quad x^2 + y^2 = 9$$

$$\underbrace{\|\omega\|_2}_3 \underbrace{\|\nu\|_2}_{17} \underbrace{\cos 45^\circ}_{\frac{\sqrt{2}}{2}} = \underbrace{\nu \cdot \omega}_{-15x + 8y}$$

$$\frac{5\sqrt{2}}{2} = -15x + 8y$$

$$x = \frac{16y - 5\sqrt{2}}{30}$$

$$\left(\frac{16y - 5\sqrt{2}}{30} \right)^2 + y^2 = 9$$

$$\frac{256y^2 - 1632\sqrt{2}y + 5202}{900} + y^2 = 9$$

$$1156y^2 - 1632\sqrt{2}y + 5202 = 8100$$

$$y = \frac{-21}{17\sqrt{2}} \pm \frac{69}{17\sqrt{2}}$$

$$\approx -.8735, 2.870$$

$$x \approx -2.8700, -.8735$$

5) (i) AV : A is 2×2 , V 2×1
to multiply, you need middle
numbers to match up, so
 VA does not make sense

$$(ii) \quad A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$$

$$V = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$a_3 = ca_1, \quad a_4 = ca_2$$

$$\begin{bmatrix} a_1 & a_2 \\ ca_1 & ca_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$a_1x + a_2y = 0$$
$$c_1x + c_2y = 0$$

↑
multiples
↓

$$a_1x + a_2y = 0$$

$$x = -a_2 \quad \text{if } A \neq \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$y = a_1$$

$$\text{if } A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$x=y=1 \quad \text{will work.}$$