Math 227 Exam 1

September 28, 2023

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.

1) a) Can you have just 1,004 solutions to a system of linear equations? Why or why not?

b) If $\vec{v}$, $\vec{t}$, and $\vec{w}$ are three nonzero vectors in $\mathbb{R}^n$, what geometric object represents $\text{span}(\vec{v}, \vec{w}, \vec{t})$?

c) For each of the following matrices in reduced row echelon form, determine whether the associated system of linear equations has solutions, and if so, how many.

(i) $\begin{bmatrix} 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 34 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 0 & 0 & 9 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

2) Find a $\boldsymbol{QUADRATIC}$ interpolating polynomial through the points $(-1, 4)$, $(2, -5)$ and $(-6, 7)$ by

a) writing down a system of linear equations that determines the coefficients of the polynomial, then

b) solving the resulting system of equations BY HAND, using any manner at your disposal and SHOWING YOUR WORK, and finally

c) writing down the polynomial.

3) Let

$$\vec{v} = \begin{bmatrix} 42 \\ -5 \\ 0 \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} 10 \\ -25 \\ 40 \end{bmatrix} \text{ and } \vec{t} = \begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix},$$

a) Write down a vector in $\text{span}(\vec{v}, \vec{w})$ that is neither a multiple of $\vec{v}$ nor a multiple of $\vec{w}$. No justification is necessary.

b) Is $\vec{t} \in \text{span}(\vec{v}, \vec{w})$? Justify your answer.

c) Can you find a vector in $\mathbb{R}^3$ that is NOT in $\text{span}(\vec{v}, \vec{w}, \vec{t})$? Why or why not?

4) Let $\mathcal{S}$ and $\mathcal{T}$ be collections of vectors in $\mathbb{R}^n$. If every vector in $\text{span}(\mathcal{S})$ is also a vector in $\text{span}(\mathcal{T})$, does this mean every vector in $\mathcal{S}$ is a vector in $\mathcal{T}$? Justify your answer.