Fall 23 Exam 1

1) a) No, only 0, 1, or infinitely many solutions

b) Either a line, plane, or 3-D space

c) (i) infinitely many

(ii) one solution

(iii) no solution

2) $y = ax^2 + bx + c$

a) $(-1, 4)$: $4 = a - b + c$

$(2, -5)$: $-5 = 4a + 2b + c$

$(-6, 7)$: $7 = 36a - 6b + c$

b) Subtract 1st two equations

$9 = -3a - 3b$

$a + b = -3$

$a = -3 - b$

Subtract 2nd two equations

$-12 = -32a + 8b$

substitute

$-12 = -32(-3 - b) + 8b$

$-12 = 96 + 40b$

$$-108 = 40b$$

$$b = -\frac{27}{10}$$

$$a = -3 - b = -3 + \frac{27}{10} = \frac{-3}{10}$$

$$c = 4 + b - a$$

$$= 4 - \frac{27}{10} + \frac{3}{10}$$

$$= \frac{40}{10} - \frac{24}{10} = 8/5$$

c) $y = \frac{-3}{10}x^2 - \frac{27}{10}x + 8/5$

b) matrix

$$\begin{array}{c} c \quad b \quad a \\ \left[\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 1 & 2 & 4 & -5 \\ 1 & -6 & 34 & 7 \end{array}\right] \end{array}$$

$R_2 - R_1$, $R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 0 & 3 & 3 & -9 \\ 0 & -5 & 35 & 3 \end{array}\right]$$

$R_2 / 3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 0 & 1 & 1 & -3 \\ 0 & -5 & 35 & 3 \end{array}\right]$$

$R1 + R2$

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & -3 \\ 0 & -5 & 35 & 3 \end{bmatrix}$$

$R_3 + 5R2$

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 40 & -12 \end{bmatrix}$$

$R_3 / 40$

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 1 & -3/10 \end{bmatrix}$$

$R2 - R3$

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & -27/10 \\ 0 & 0 & 1 & -3/10 \end{bmatrix}$$

$R1 - 2R3$

$$\begin{bmatrix} 1 & 0 & 0 & 8/5 \\ 0 & 1 & 0 & -27/10 \\ 0 & 0 & 1 & -3/10 \end{bmatrix}$$

3) a) $\vec{v}+\vec{w} = \begin{bmatrix} 52 \\ -30 \\ 40 \end{bmatrix}$

b) $c\vec{v} + d\vec{w} = \vec{t}$

$$c\begin{bmatrix} 42 \\ -5 \\ 0 \end{bmatrix} + d\begin{bmatrix} 10 \\ -25 \\ 40 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix}$$

$40d = 1$

$d = 1/40$

$-5c - 25d = 0$

$c + 5d = 0$

$c + \frac{1}{8} = 0$

$c = -1/8$

check: $\frac{-1}{8} \cdot 42 + \frac{1}{40} \cdot 10$

$= -\frac{21}{4} + \frac{1}{4} = \frac{-20}{4} = 5$ ✓

c) Yes - since $\vec{t} \in \text{span}(\vec{v}, \vec{w})$,

$$\text{span}(\vec{t}, \vec{v}, \vec{w}) = \text{span}(\vec{v}, \vec{w}).$$

Since $\vec{v}$ and $\vec{w}$ aren't multiples, their span is a plane, not all of 3D space.

4) No. Let $S = \left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\}$, $T = \left\{\begin{bmatrix} 2 \\ 0 \end{bmatrix}\right\}$

Then $\text{span}(S) = \text{span}(T)$, but

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 2 \\ 0 \end{bmatrix},$$

so this isn't true.