Name:

Math 227 Exam 1

February 6, 2013

Directions: WRITE YOUR NAME ON THIS TEST! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. DO NOT convert irrational numbers such as $\sqrt{3}$ or π into decimal approximations; just leave them as they are.

1) (2 points each) True/False. If the sentence is false, correct the error. No justification is necessary.

a) The transpose A^t of a matrix A is only defined for $n \times n$ matrices.

b) If a system of linear equations has two different solutions, then it has infinitely many solutions.

c) If none of the vectors v_1, v_2, and v_3 are multiples of one of the other vectors, then it is always true that $\{v_1, v_2, v_3\}$ is linearly independent.

d) For a vector v in $\mathbb{R}^n$, $\|v\|_2 = v \cdot v$.

e) If A is a 2×3 matrix and B is a 5×3 matrix, then the matrix product AB makes sense.

2) Aluminum (Al) combines with oxygen (O) to produce aluminum oxide (Al_2O_3) via the equation

$$Al + O_2 \to Al_2O_3.$$

a) (10 points) Determine a system of linear equations (or a matrix) that balances the equation.

b) (10 points) Balance the equation. *Note:* If you can do this without using part a), you will get full credit for the entire problem.

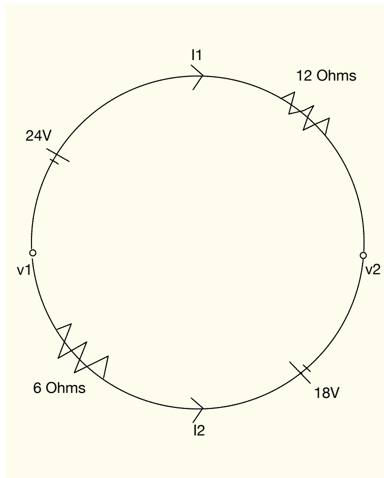
3) Let $A = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -1 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & -10 & 4 \\ 12 & 9 & -8 \end{bmatrix}$.

a) (8 points) Compute the matrix product AB and find $(AB)^t$.

b) (10 points) Compute A^t, B^t, and B^tA^t.

c) (2 points) Compare your answers for $(AB)^t$ and B^tA^t. What do you find?

4) Consider the following electrical circuit:



a) (4 points) Find the edge-node incidence matrix A.

b) (4 points) Determine the resistance matrix R.

c) (10 points) Set up a matrix equation for finding the currents I_1 and I_2 and the potential differences between v_1 and v_2.

d) (8 points) Find the currents and potential differences.

5) a) (6 points) If $v_1, v_2, \ldots, v_n, v$ are vectors in $\mathbb{R}^m$, define what it means for v to be in $Span\{v_1.v_2, \ldots, v_n\}$.

b) (8 points) Let

$$v_1 = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}, \ v_2 = \begin{bmatrix} 3 \\ 10 \\ -4 \end{bmatrix}, \ v_3 = \begin{bmatrix} 9 \\ -8 \\ 15 \end{bmatrix} \text{ and } v = \begin{bmatrix} 39 \\ -48 \\ 52 \end{bmatrix}.$$

Show that v is in $Span\{v_1, v_2, v_3\}$.

b) (10 points) For all nonzero vectors w_1, w_2 and w_3 in $\mathbb{R}^2$ with w_3 orthogonal to both w_1 and w_2, show that $w_1 + w_3$ is linearly independent of w_2.

BONUS: (10 points) Let $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. For all 2×2 matrices $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ with $ad - bc \neq 0$, the vectors Ae_1 and Ae_2 determine a parallelogram. Find the area of the parallelogram, with work to support your assertion.