Name:

Math 227 Exam 1

February 9, 2015

Directions: WRITE YOUR NAME ON THIS TEST! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. Decimal approximations, accurate to four decimal places, are acceptable.

1) Vanadium oxide combines with hydrogen chloride to produce vanadium oxytrichloride and water via

$$V_2O_5 + HCl \to VOCl_3 + H_2O$$

a) (6 points) Determine a system of linear equations (or a matrix) that balances the equation.

b) (8 points) Balance the equation. *Note:* If you can do this without using part a), you will get full credit for the entire problem, but I will need to see some work.

2) Find a quadratic interpolating polynomial through the points $(1, 5)$, $(3, 2)$, and $(-1, 6)$ by

a) (6 points) writing down a system of linear equations that determines the coefficients of the polynomial, then

b) (8 points) producing an augmented matrix for the system and row-reducing it, and finally

c) (3 points) writing down the polynomial.

3) Define the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$,

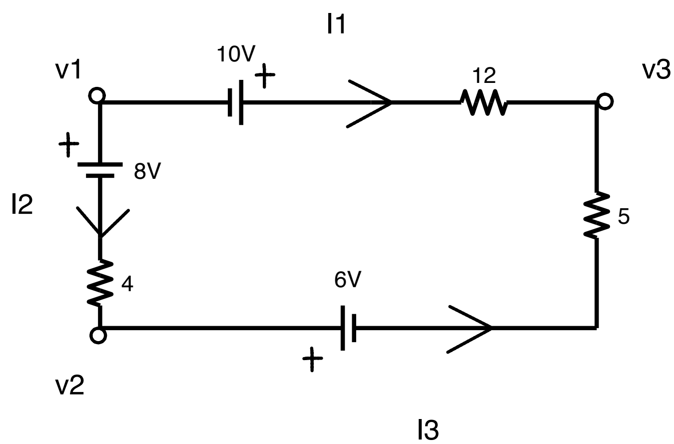
$$T(x, y) = \left(\frac{x+y}{\sqrt{2}}, \frac{x-y}{\sqrt{2}} \right).$$

a) (7 points) Show that T is linear.

b) (4 points) Find the standard matrix A of T.

c) (8 points) Show that $T(T(x, y)) = (x, y)$.

4) Consider the following electrical circuit (resistance is in Ohms):



a) (4 points) Find the edge-node incidence matrix A.

b) (4 points) Determine the resistance matrix R.

c) (6 points) Set up a matrix equation for finding the currents I_1, I_2, and I_3 and the potential differences between v_1, v_2, and v_3.

d) (8 points) Find the currents and potential differences.

5) a) (8 points) Let

$$v_1 = \begin{bmatrix} 2 \\ 3 \\ -1 \\ 6 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 8 \\ -4 \\ 10 \\ 7 \end{bmatrix}$$

Find a vector v in $span\{v_1, v_2\}$ that is neither a scalar multiple of v_1 nor a scalar multiple of v_2. Then find a vector w that is NOT in $span\{v_1, v_2\}$. Be sure to show your answers are correct.

b) (10 points) For all linearly independent vectors $v_1, v_2,$, and v_3 in $\mathbb{R}^n$, prove that $\{v_1, v_1 + 2v_2, v_1 + 2v_2 + 3v_3\}$ is also linearly independent.