Name:

Math 227 Exam 1

February 15, 2018

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.

1) (15 points) Solve the following system of equations BY HAND, using any manner at your disposal, or show there is no solution. SHOW YOUR WORK.

$$x + y = -3$$
$$2x + y + 4z = 5$$
$$-2x + y - 3z = 0.$$

2) Find a cubic interpolating polynomial through the points $(1, 5)$, $(-1, 3)$, $(2, 4)$ and $(-2, 7)$ by

a) (8 points) writing down a system of linear equations that determines the coefficients of the polynomial, then

b) (8 points) producing an augmented matrix for the system and row-reducing it, and finally

c) (4 points) writing down the polynomial.

3) By averaging over the indicated blocks, "compress" the following matrix

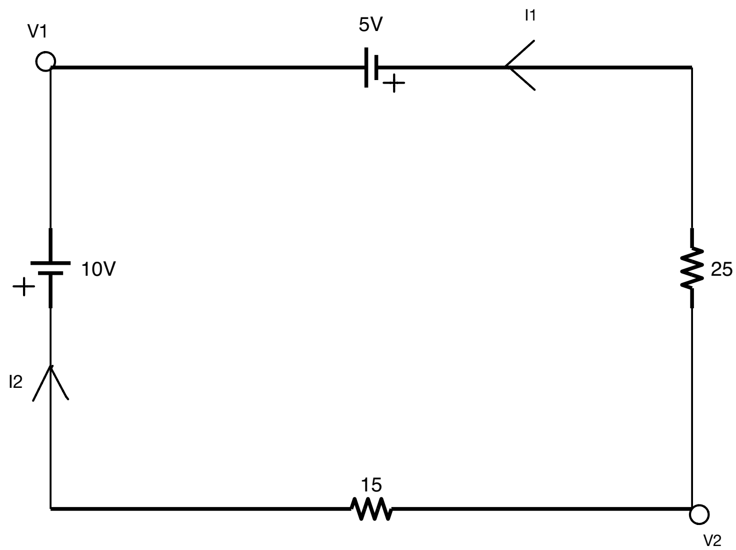
$$A = \left[\begin{array}{c|c} 13 & 16 \\ -2 & 24 \\ \cline{2-2} -7 & 5 \end{array}\right]$$

by

a) (8 points) vectorizing the matrix, writing down the appropriate matrices involved, then

b) (6 points) multiplying by an appropriate matrix to average the vector.

4) Consider the following electrical circuit (resistance is in Ohms):



a) (4 points) Find the edge-node incidence matrix A.

b) (4 points) Determine the resistance matrix R.

c) (6 points) Set up a matrix equation for finding the currents I_1 and I_2 and the potential differences between v_1 and v_2.

d) (6 points) Find the currents and potential difference.

5) a) (3 points) Let

$$v_1 = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

Find a nonzero vector w that is orthogonal to v_1.

b) (6 points) Let

$$v_2 = \begin{bmatrix} 5 \\ 1 \\ -2 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix}$$

Find TWO vectors w_1 and w_2 in span$\{v_2, v_3\}$ that are neither a scalar multiple of v_2, nor a scalar multiple of v_3, nor a scalar multiple of eachother.

c) (8 points) With v_2 and v_3 as in part b), find a nonzero vector t that is orthogonal to both v_2 and v_3. Be sure to show your answer is correct.

6) a) (4 points) Define linear independence of a collection of vectors $\{v_1, v_2, \ldots, v_k\}$ in $\mathbb{R}^n$.

b) (10 points) For ALL nonzero vectors v and w in $\mathbb{R}^2$ and 2×2 matrices A with real entries, show that if $A(v)$ and $A(w)$ are not parallel then v and w are linearly independent.

BONUS: (10 points) For ALL nonzero, nonparallel vectors v_1 and v_2 in $\mathbb{R}^n$, let w and v be two ARBITRARY (i.e. you cannot prescribe a choice) nonzero, nonparallel vectors in $span\{v_1, v_2\}$ Let u be a nonzero vector that is orthogonal to both w and v. Show that u is orthogonal to both v_1 and v_2.