# Math 227 Exam 1

## February 6, 2020

**Directions:**

1. WRITE YOUR NAME ON THIS TEST!

2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.

3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.

4. If you have a question, raise your hand or come up and ask me.

**1)** a) (4 points) Can you have just two solutions to a system of linear equations? Why or why not?

b) (6 points) For each of the following matrices in row-reduced echelon form, determine whether the associated system of linear equations has solutions, and if so, how many.

(i) $\begin{bmatrix} 1 & 0 & 5 & 6 & 1 \\ 0 & 1 & 0 & 7 & -2 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 0 & 0 & 9 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

**2)** Determine whether the following computations can be effected. If the computation cannot be done, briefly explain why. If the computation can be done, give the dimensions of the resulting output.

$$A = \begin{bmatrix} -1 & 1 \\ 1 & 0 \\ 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & 3 \\ -1 & 2 & 1 \\ 0 & 3 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 4 & -2 \\ 2 & 0 & 1 \end{bmatrix} \quad v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

(a) (3 points)  $Bv$

(b) (3 points)  $vB$

(c) (3 points)  $AB$

(d) (3 points)  $AC$

(e) (3 points)  $A^T + C$

**3)** Find a ***QUADRATIC*** interpolating polynomial through the points $(-2, 5)$, $(0, 6)$ and $(1, 8)$ by

a) (10 points) writing down a system of linear equations that determines the coefficients of the polynomial, then

b) (15 points) solving the resulting system of equations BY HAND, using any manner at your disposal and SHOWING YOUR WORK, and finally

c) (4 points) writing down the polynomial.

**4)** For vectors

$$v = \begin{bmatrix} 4 \\ 5 \\ -2 \end{bmatrix}, \quad w = \begin{bmatrix} -10 \\ 7 \\ 42 \end{bmatrix}, \quad \text{and } u = \begin{bmatrix} -61 \\ 31 \\ 234 \end{bmatrix},$$

a) (10 points) Compute $\|v\|_2$ and $w \cdot u$.

b) (15 points) Find numbers $x$ and $y$ with $xv + yw - u = \vec{0}$.

c) (5 points) What geometric object is formed by all linear combinations of $v, w,$ and $u$? Why?

**5)** a) (8 points) For ALL arbitrary $2 \times 2$ matrices $A$, show that if $Ax = \vec{0}$ for a *nonzero* vector $x$, then $\det(A) = 0$.

b) (8 points) For ALL $P$ and $Q$ arbitrary $4 \times 4$ permutation matrices (that is, matrices that have the same rows as $I_4$ but potentially in a different order), show that $P - Q$ is not invertible by finding a *nonzero* $x$ with $(P - Q)x = \vec{0}$.