1) a) no; you can only have 0, 1, or infinitely many solutions

b) (i) infinitely many solutions

(ii) no solution

(iii) unique solution

2) a) yes, 3×1

b) no $(3 \times 1) \times (3 \times 3)$

not equal

c) no $(3 \times 2) \times (3 \times 3)$

not equal

d) yes, 3×3

e) yes, 2×3

3) a) $y = ax^2 + bx + c$

$5 = 4a - 2b + c$

$6 = c$

$8 = a + b + c$

b) Substitute $c = 6$

$5 = 4a - 2b + 6$

$8 = a + b + 6$

so $-1 = 4a - 2b$

$2 = a + b$

$$\begin{bmatrix} 1 & 1 & 2 \\ 4 & -2 & -1 \end{bmatrix}$$

$-4R1 + R2$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -6 & -9 \end{bmatrix}$$

$1/6 \, R2$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -1 & -3/2 \end{bmatrix}$$

$R2 + R1$

$$\begin{bmatrix} 1 & 0 & 1/2 \\ 0 & -1 & -3/2 \end{bmatrix}$$

$-R2$

$$\begin{array}{c} \begin{array}{ccc} a & b & \end{array} \\ \begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 3/2 \end{bmatrix} \end{array}$$

$$y = \frac{1}{2}x^2 + \frac{3}{2}x + 6$$

4) a) $\|v\|_2 = \sqrt{16+25+4}$

$= \sqrt{45} = 3\sqrt{5}$

$w \cdot u = (-10)(-61) + 7(31) + (42)(234)$

$= 610 + 217 + 9828$

$= 10655$

b) $x\begin{bmatrix} 4 \\ 5 \\ -2 \end{bmatrix} + y\begin{bmatrix} -10 \\ 7 \\ 42 \end{bmatrix} = \begin{bmatrix} -61 \\ 31 \\ 234 \end{bmatrix}$

$\begin{bmatrix} 4 & -10 \\ 5 & 7 \\ -2 & 42 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -61 \\ 31 \\ 234 \end{bmatrix}$

$\begin{bmatrix} 4 & -10 & -61 \\ 5 & 7 & 31 \\ -2 & 42 & 234 \end{bmatrix}$

$$\begin{bmatrix} -2 & 42 & 234 \\ 4 & -10 & -61 \\ 5 & 7 & 31 \end{bmatrix}$$

$R1/-2$

$$\begin{bmatrix} 1 & -21 & -117 \\ 4 & -10 & -61 \\ 5 & 7 & 31 \end{bmatrix}$$

$-4R1+R2$

$$\begin{bmatrix} 1 & -21 & -117 \\ 0 & 74 & 407 \\ 5 & 7 & 31 \end{bmatrix}$$

$-5R1+R3$

$$\begin{bmatrix} 1 & -21 & -117 \\ 0 & 74 & 407 \\ 0 & 112 & 616 \end{bmatrix}$$

$R2/74$

$$\begin{bmatrix} 1 & -21 & -117 \\ 0 & 1 & 11/2 \\ 0 & 112 & 616 \end{bmatrix}$$

$21R_2 + R_1$

$$\begin{bmatrix} 1 & 0 & -3/2 \\ 0 & 1 & 11/2 \\ 0 & 1/2 & 6/6 \end{bmatrix}$$

$-1/2 R_2 + R_3$

$$\begin{bmatrix} 1 & 0 & -3/2 \\ 0 & 1 & 11/2 \\ 0 & 0 & 0 \end{bmatrix}$$

$x = -3/2$

$y = 11/2$

c) a plane, since v and w are not scalar multiples, all their linear combinations span a plane, and part b) shows u is in that plane.

5) a) Suppose $Ax = 0$. Then if A had nonzero determinant, A would be invertible, and then applying A^{-1} to both sides,

$$x = A^{-1}(Ax) = A^{-1}(0) = 0$$

b) $$\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = 1$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = 1$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = 1$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = 1$$

Then this shows that, no matter what order the rows go in,

$$P \begin{bmatrix} | \\ | \\ | \\ | \end{bmatrix} = \begin{bmatrix} | \\ | \\ | \\ | \end{bmatrix}$$

and

$$Q \begin{bmatrix} | \\ | \\ | \\ | \end{bmatrix} = \begin{bmatrix} | \\ | \\ | \\ | \end{bmatrix}$$

$$\text{so } (P-Q) \begin{bmatrix} | \\ | \\ | \\ | \end{bmatrix} = P \begin{bmatrix} | \\ | \\ | \\ | \end{bmatrix} - Q \begin{bmatrix} | \\ | \\ | \\ | \end{bmatrix}$$

$$= \begin{bmatrix} | \\ | \\ | \\ | \end{bmatrix} - \begin{bmatrix} | \\ | \\ | \\ | \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \checkmark$$