Math 227 Exam 1

February 10, 2022

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.

1) a) Can you have just 402 solutions to a system of linear equations? Why or why not?

b) Each of the following matrices represents the augmented matrix to a system of linear equations and is in reduced row echelon form. Determine whether the associated system of linear equations has solutions, and if so, how many.

(i) $\begin{bmatrix} 1 & 0 & 4 & -17 \\ 0 & 1 & 6 & 0 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 0 & 0 & 15 \\ 0 & 1 & 0 & \pi \\ 0 & 0 & 1 & -\sqrt{34} \\ 0 & 0 & 0 & 0 \end{bmatrix}$

c) (9 points) Write down the 3×3 matrix $A = (A_{i,k})$ where

$$A_{i,k} = 2^k - 5i$$

for all $1 \leq i, k \leq 3$.

2) Let

$$A = \begin{bmatrix} 5 & -9 \\ \sqrt{2} & -8 \end{bmatrix}, \quad B = \begin{bmatrix} 8 & 3 \\ 2 & -4 \\ 3 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 4 \\ 2 & 0 \\ 11 & -5 \\ -12 & 10 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 1 \\ 4 \\ -7 \\ 13 \end{bmatrix}.$$

(a) What are the dimensions of the matrices A, B, C, and $\vec{v}$?

(b) Determine whether the following computations can be effected. If the computation can be done, give the dimensions of the resulting output.

(i) $\vec{v}^t \cdot A$

(ii) $C^t \cdot \vec{v}$

(iii) $A \cdot C$

(iv) $B \cdot C$

3) Find a **QUADRATIC** interpolating polynomial through the points $(1, -2)$, $(2, 5)$ and $(6, -3)$ by

a) writing down a system of linear equations that determines the coefficients of the polynomial, then

b) solving the resulting system of equations BY HAND, using any manner at your disposal and SHOWING YOUR WORK, and finally

c) writing down the polynomial.

4) For vectors

$$\vec{v} = \begin{bmatrix} 8 \\ 0 \\ 7 \end{bmatrix}, \ \vec{w} = \begin{bmatrix} 2 \\ -10 \\ 3 \end{bmatrix}, \text{ and } \vec{u} = \begin{bmatrix} -7 \\ 3 \\ 8 \end{bmatrix},$$

a) Compute $\vec{v}^t \cdot \vec{u}$.

b) What geometric object represents $\text{span}(\vec{v}, \vec{w})$? Justify your answer.

c) Write down a vector in $\text{span}(\vec{v}, \vec{w})$ that is neither a multiple of $\vec{v}$ nor a multiple of $\vec{w}$. No justification is necessary.

d) Is there a vector in $\mathbb{R}^3$ that is NOT in $\text{span}(\vec{v}, \vec{w}, \vec{u})$? Justify your answer.

5) a) For $x \in \mathbb{R}$, if $x^2 = 0$, then $x = 0$. Find a NONZERO 2×2 matrix A with $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
.

b) Show, via a choice of 2×2 matrices A, B, and C with A NONZERO that it is possible to have $AB = AC$, but $B \neq C$. This shows that, in general, you cannot "cancel" matrix multiplication.