Math 227 Exam 1

February 6, 2023

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.

1) a) Can you have just 892 solutions to a system of linear equations? Why or why not?

b) Write down the 3×3 matrix $A = (A_{i,j})$ where

$$A_{i,j} = (3j - 4)^i$$

for all $1 \leq i, j \leq 3$.

2) Let

$$A = \begin{bmatrix} 5 & -9 \\ \sqrt{2} & -8 \end{bmatrix}, \quad B = \begin{bmatrix} 8 & 3 \\ 2 & -4 \\ 3 & 1 \\ -12 & 10 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 4 \\ 2 & 0 \\ 11 & -5 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 3 \\ -6 \\ 1 \end{bmatrix}.$$

(a) What are the dimensions of the matrices A, B, C, and $\vec{v}$?

(b) Determine whether the following computations can be effected. If the computation can be done, give the dimensions of the resulting output. If the computation cannot be done, merely say so.

(i) $B^t \cdot \vec{v}$

(ii) $\vec{v}^t \cdot C$

(iii) $A^t \cdot C^t$

(iv) $C \cdot B^t$

3) Find a **QUADRATIC** interpolating polynomial through the points $(1, 3)$, $(-4, -7)$ and $(-5, -2)$ by

a) writing down a system of linear equations that determines the coefficients of the polynomial, then

b) solving the resulting system of equations BY HAND, using any manner at your disposal and SHOWING YOUR WORK, and finally

c) writing down the polynomial.

4) For vectors

$$\vec{v} = \begin{bmatrix} -42 \\ -42 \end{bmatrix} \text{ and } \vec{w} = \begin{bmatrix} -42 \\ 42 \end{bmatrix},$$

a) What geometric object represents $\text{span}(\vec{v})$?

b) Write down a vector in $\text{span}(\vec{v}, \vec{w})$ that is neither a multiple of $\vec{v}$ nor a multiple of $\vec{w}$. No justification is necessary.

c) Find a vector of length 5 whose angle between both $\vec{v}$ and $\vec{w}$ is $45°$.

d) Is there a vector in $\mathbb{R}^2$ that is NOT in $\text{span}(\vec{v}, \vec{w})$? Justify your answer.

5) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a 2×2 matrix.

a) If $A^2 = A$, show $a + d$ is either 0, 1, or 2.

b) If $A^t A\vec{v} = \vec{0}$ for some vector $\vec{v}$, show that $A\vec{v} = \vec{0}$.