Name:

Math 227 Exam 2

November 4, 2021

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.

1) Let V, W be a vector spaces. Let $T : V \to W$ be a linear function.

a) (4 points) What are the two operations on V, i.e., what makes a vector space?

b) (3 points) Let $V = \mathcal{S}(\mathbb{R})$. What are the vectors?

c) (3 points) What are the possible geometric descriptions of subspaces of $\mathbb{R}^2$?

d) (4 points) What are the possible geometric descriptions of subspaces of $\mathbb{R}^3$?

e) (4 points) If A is an $n \times n$ matrix and A is invertible, what do you know about $\det(A)$?

2) Find a single 3×3 matrix that, in homogeneous coordinates,

a) (8 points) rotates a 2-vector by $3\pi/2$ radians clockwise,

b) (6 points) scales the x-coordinate of a 2-vector up by a factor of 42 and scales the y-coordinate down by a factor of 3.

c) (7 points) shifts a 2-vector down 4 units and left 10 units,

d) (6 points) does a)-c) in order, starting with a).

3) a) (2 points, Fill-in-the-blank) Every linear function from $\mathbb{R}^n$ to $\mathbb{R}^m$ is given by a ____________.

b) (13 points) Let $T : \mathbb{R}^4 \to \mathbb{R}^3$,

$$T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} z - 3x + 2y \\ x + z \\ 0 \end{bmatrix}.$$

Show that T is linear.

4) (20 points) Let

$$W = \{f \in \mathcal{F}(\mathbb{R}) \mid (f(12) - f(7)) \in \mathbb{R}\}.$$

Show that W is a subspace of $\mathcal{F}(\mathbb{R})$.

5) (20 points) Let

$$S = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \mid (x-z)^4 = (y-z)^4 \right\}.$$

Show that S is NOT a subspace of $\mathbb{R}^3$.