Name:

Math 227 Exam 2

November 7, 2018

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Wolfram Alpha or a similar program may be used for all computational problems.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.

1) Let V be a vector space.

a) (4 points) What are the two operations on V, i.e., what makes a vector space?

b) (2 points) If $V = M_3(\mathbb{R})$, what are the vectors?

c) (2 points) If $V = \mathbb{P}[x]$, what are the vectors?

d) (2 points) If v and w are column vectors in $\mathbb{R}^n$, what is the difference between the quantities $v \cdot w$ and $v^t w$?

e) (3 points) Fill in the blank: If A, $B \in M_n(\mathbb{R})$, $(AB)^t =$ ____________.

2) Find a single 3×3 matrix that, in homogeneous coordinates,

a) (4 points) rotates a 2-vector by $2\pi/3$ radians counterclockwise,

b) (8 points) scales a 2-vector down by a factor of 8,

c) (6 points) shifts a 2-vector up 10 units and right 3 units,

d) (6 points) does a)-c) in order, starting with a).

3) Let T be the linear transformation from $\mathbb{R}^4 \to \mathbb{R}$ given by

$$T\left(\begin{bmatrix} x \\ y \\ w \\ z \end{bmatrix}\right) = 13x - 52y - 18z + 4w.$$

a) (10 points) Show that T is linear.

b) (7 points) Calculate the orthogonal projection onto $\mathrm{Ran}(T)$.

4) Given the points $(-1, 3)$, $(8, 7)$, $(6, -2)$ and $(-4, -4)$ in $\mathbb{R}^2$, find the best-fit quadratic to the points by

a) (12 points) Finding a system of linear equations that represents a "solution" to the problem,

b) (6 points) Writing the problem as a matrix equation $Ax = b$,

c) (6 points) Finding the system $A^t Ax = A^t b$, computing both $A^t A$ and $A^t b$,

d) (4 points) Solving the system in c) and producing the polynomial.

5) a) (10 points) Let

$$W = \{(x, y, z) \in \mathbb{R}^3 \mid 2^{x+y+z} = 1\}.$$

Show that W is a subspace of $\mathbb{R}^3$.

b) (8 points) Let $S = \{A \in M_2(\mathbb{R}) \mid \det(A) \geq 0\}$. Show that S is NOT a subspace of $M_2(\mathbb{R})$.

BONUS: (10 points) Let V and W be vector spaces and $T : V \to W$ a linear transformation. Show that $\mathrm{Ran}(T)$ is a subspace of W.