Exam 2 Fall '18

10:02

1 a) addition & scalar multiplication

b) 3x3 matrice

c) polynomials with real coefficients

d) none

e) $B^t A^t$

2 a)

$$\begin{bmatrix} \cos(2\pi/3) & -\sin(2\pi/3) & 0 \\ \sin(2\pi/3) & \cos(2\pi/3) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b)

$$\begin{bmatrix} 1/8 & 0 & 0 \\ 0 & 1/8 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

c) $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} -\frac{1}{16} & \frac{\sqrt{3}}{16} & 3 \\ \frac{\sqrt{3}}{16} & -\frac{1}{16} & 10 \\ 0 & 0 & 1 \end{bmatrix}$

3) a)

$$T\left(\begin{bmatrix}1\\0\\0\\0\end{bmatrix}\right) = 13$$

$$T\left(\begin{bmatrix}0\\1\\0\\0\end{bmatrix}\right) = -52$$

$$T\left(\begin{bmatrix}0\\0\\1\\0\end{bmatrix}\right) = -18$$

$$T\left(\begin{bmatrix}0\\0\\0\\1\end{bmatrix}\right) = 4$$

matrix for T $\begin{bmatrix}13 & -52 & -18 & 4\end{bmatrix}$

so T is linear

— or —

Let $c \in \mathbb{R}$, $\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ w_1 \end{bmatrix}, \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ w_2 \end{bmatrix} \in \mathbb{R}^4$

$$T\left(c\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ w_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ w_2 \end{bmatrix}\right)$$

$$= T\left(\begin{bmatrix} cx_1 \\ cy_1 \\ cz_1 \\ cw_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ w_2 \end{bmatrix}\right)$$

$$= T\left(\begin{bmatrix} cx_1 + x_2 \\ cy_1 + y_2 \\ cz_1 + z_2 \\ cw_1 + w_2 \end{bmatrix}\right)$$

$$= 13(cx_1 + x_2) - 52(cy_1 + y_2) - 18(cz_1 + z_2)$$
$$+ 4(cw_1 + w_2)$$

$+1$

$$= 13cx_1 - 52cy_1 - 18cz_1 + 4cw_1$$

$$+ 13x_2 - 52y_2 - 18z_2 + 4w_2$$

$$= c(13x_1 - 52y_1 - 18z_1 + 4w_1)$$

$$+ 13x_2 - 52y_2 - 18z_2 + 4w_2$$

$$= cT\left(\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ w_1 \end{bmatrix}\right) + T\left(\begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ w_2 \end{bmatrix}\right)$$

b) $T: \mathbb{R}^4 \to \mathbb{R}$ and

$T \neq 0$, so $\text{Ran}(T) = \mathbb{R}$

orthogonal projection $= 1$

a) $a - b + c = 3$

$64a + 8b + c = 7$

$36a + 6b + c = -2$

$16a - 4b + c = -4$

b) $$\begin{bmatrix} 1 & -1 & 1 \\ 64 & 8 & 1 \\ 36 & 6 & 1 \\ 16 & -4 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -2 \\ 4 \end{bmatrix}$$

c) $$A^tA = \begin{bmatrix} 5649 & 663 & 117 \\ 663 & 117 & 9 \\ 117 & 9 & 4 \end{bmatrix}$$

$$A^t b = \begin{bmatrix} 315 \\ 57 \\ 4 \end{bmatrix}$$

d) $$\begin{bmatrix} -\frac{7}{12364} \\ \frac{6163}{12364} \\ -\frac{59}{562} \end{bmatrix} = (A^t A)^{-1} A^t b$$

$$-\frac{7}{12364} x^2 + \frac{6163}{12364} x - \frac{59}{562} = y$$

$$-.000566 x^2 + .498463 x - .104982 = y$$

5) a) $2^{x+y+z} = 1$

$\Leftrightarrow \ln(2^{x+y+z}) = \ln(1) = 0$

$\Leftrightarrow (x+y+z)\ln(2) = 0$

$x+y+z = 0$

Let $A = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$

$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = x+y+z$, so

$W = \ker(A)$, all

Kernels of matrices are subspaces

— or —

$$2^{(0+0+0)} = 2^0 = 1$$

so $(0,0,0) \in W$

Now let $(x_1, y_1, z_1), (x_2, y_2, z_2) \in W$

$c \in \mathbb{R}$

Then $2^{x_1+y_1+z_1} = 1$, $2^{x_2+y_2+z_2} = 1$

$$2^{c(x_1+y_1+z_1)+x_2+y_2+z_2}$$

$$= 2^{c(x_1+y_1+z_1)} \cdot 2^{x_2+y_2+z_2}$$

$$= \left(2^{x_1+y_1+z_1}\right)^c \cdot 2^{x_2+y_2+z_2} = 1^c \cdot 1^c$$

$$= 1 \checkmark$$

b) Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $\det(A) = 0$

$B = \begin{bmatrix} 0 & 6 \\ 0 & 0 \end{bmatrix}$, $\det(B) = 0$

$A + B = \begin{bmatrix} 1 & 7 \\ 1 & 1 \end{bmatrix}$

$\det(A+B) = 1 - 7 = -6 < 0$

so $A + B \notin S$, S

is not a subspace