Name:

Math 227 Exam 2

November 5, 2019

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Wolfram Alpha or a similar program may be used for all computational problems.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.

1) Let V be a vector space.

a) (4 points) What are the two operations on V, i.e., what makes a vector space?

b) (3 points) If $V = M_{2\times 3}(\mathbb{R})$, what are the vectors?

c) (3 points) If $A \in M_{2\times 3}(\mathbb{R})$, $B \in M_{4\times 2}(\mathbb{R})$, only one of AB or BA makes sense. Which is it, and what are the dimensions of the resulting product?

d) (3 points) Fill in the blank: If $A \in M_n(\mathbb{R})$ is a non-invertible matrix, then

$$\det(A) \underline{\hspace{3cm}}.$$

e) (3 points) Fill in the blank: If A, $B \in M_n(\mathbb{R})$ are invertible,

$$(AB)^{-1} = \underline{\hspace{3cm}}.$$

2) Find a single 3×3 matrix that, in homogeneous coordinates,

a) (8 points) rotates a 2-vector by $2\pi/3$ radians counterclockwise,

b) (4 points) scales a 2-vector down by a factor of 8,

c) (6 points) shifts a 2-vector up 10 units and right 3 units,

d) (6 points) does a)-c) in order, starting with a).

3) a) (10 points) For ALL $n \times n$ matrices A such that A^3 is invertible, show that A is also invertible.

b) (12 points) Via a choice of A and B in $M_2(\mathbb{R})$, show that, in general,

$$(A + B)^2 \neq A^2 + 2AB + B^2.$$

4) (20 points) Let

$$W = \{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_2(\mathbb{R}) \mid b + d = a - c \}.$$

Show that W is a subspace of $M_2(\mathbb{R})$.

5) (18 points) Let

$$S = \{(x, y, z, w) \in \mathbb{R}^4 \mid \sqrt{x^2 + y^2} = \sqrt{z^2 + w^2}\}.$$

Show that S is NOT a subspace of $\mathbb{R}^4$.