Exam 2 Fall '19

1) a) addition and scalar multiplication

b) 2×3 matrices

c) BA, 4×3

d) $= 0$

e) $B^{-1}A^{-1}$

2)

a) $\begin{bmatrix} \cos(2\pi/3) & -\sin(2\pi/3) & 0 \\ \sin(2\pi/3) & \cos(2\pi/3) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = A$

b) $\begin{bmatrix} 1/8 & 0 & 0 \\ 0 & 1/8 & 0 \\ 0 & 0 & 1 \end{bmatrix} = B$

c) $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix} = C$

a) $C \cdot B \cdot A$

$$= \begin{bmatrix} -\frac{1}{16} & -\frac{\sqrt{3}}{16} & 3 \\ \frac{\sqrt{3}}{16} & -\frac{1}{16} & 10 \\ 0 & 0 & 1 \end{bmatrix}$$

3) a) $0 \neq \det(A^3)$

$= \det(A \cdot A \cdot A)$

$= (\det(A))^3$

Taking cube roots,

$\det(A) \neq 0$, and so A is

invertible.

b) Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$A+B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$, $(A+B)^2 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$A^2 + 2AB + B^2 = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \neq \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

4) i) zero vector $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A_{1,2} + A_{2,2} = 0+0=0$

$A_{1,1} - A_{2,1} = 0+0=0$, so the zero vector is in W ✓

Now let $A, B \in W$, $c \in \mathbb{R}$.

ii) Scalar multiplication:

$$cA = (cA_{i,j}), \text{ so}$$

$$(cA)_{1,2} + (cA)_{2,2} = cA_{1,2} + cA_{2,2}$$

$$= c(A_{1,2} + A_{2,2})$$

$$= c(A_{1,1} - A_{2,1})$$

$$= cA_{1,1} - cA_{2,1}$$

$$= (cA)_{1,1} - (cA)_{2,1}$$

✓

(iii) Addition

$$A+B = (A_{i,j} + B_{i,j}), \text{ so}$$

$$(A+B)_{1,2} + (A+B)_{2,2} = A_{1,2} + B_{1,2} + A_{2,2} + B_{2,2}$$

$$= A_{1,2} + A_{2,2} + B_{1,2} + B_{2,2}$$

$$= A_{1,1} - A_{2,1} + B_{1,1} - B_{2,1}$$

$$= (A_{1,1} + B_{1,1}) - (A_{2,1} + B_{2,1})$$

$$= (A+B)_{1,1} - (A+B)_{2,1}$$

5)

zero vector: $0^2+0^2 = 0^2+0^2$

$0 = 0$

does not fail

scalars: $c(x,y,z,w) = (cx, cy, cz, cw)$

$$\sqrt{(cx)^2+(cy)^2} = \sqrt{c^2x^2+c^2y^2}$$

$$= \sqrt{c^2(x^2+y^2)}$$

$$= |c|\sqrt{x^2+y^2}$$

$$= |c|\sqrt{z^2+w^2}$$

$$= \sqrt{(cz)^2+(cw)^2}$$

does not fail

addition: $v = (1,0,0,1) \in S$

$w = (-1,0,0,1) \in S$

but $v+w = (0,0,0,2)$

$0 \neq 2$, so $v+w \notin S$.