Name:

Math 227 Exam 2

October 25, 2023

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.

1) Let V, W be vector spaces.

a) What are the two operations on V, i.e., what makes a vector space?

b) If $V = M_4(\mathbb{R})$, what are the vectors in V?

c) If an $n \times n$ matrix A does not have its determinant equal to zero, what does this tell you regarding the invertibility of A?

d) If A is a 2×3 matrix, B is a 3×1 matrix, and C is a 3×2 matrix, write down whether the following operations are possible or not. No justification is necessary.

i) $A \cdot B$

ii) $C^t \cdot B$

iii) $A + C$

iv) $A + C^t$

2) Find a single 3×3 matrix that, in homogeneous coordinates,

a) scales the x-coordinate of a 2-vector down by a factor of 8 and scales the y-coordinate up by a factor of 7,

b) rotates a 2-vector by $2\pi/3$ radians clockwise,

c) shifts a 2-vector up 10 units and right 6 units.

d) If A, B, and C are the matrices from parts a), b), and c), respectively, in what order do you write the product of A, B, and C if you first scale, then shift, then rotate?

3) a) Calculate the area of the parallelogram with vertices $(0, 0)$, $(-3, 1)$, $(4, 2)$, and $(1, 3)$. Be sure to draw a picture!

b) If $\vec{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, find a vector of length 5 whose angle with $\vec{v}$ is $30°$.

4) Recall that $\mathcal{S}$ is the vector space of all sequences of real numbers. Let

$$W = \{(a_n) \in \mathcal{S} \ : \ \sum_{n=1}^{\infty} a_n = 0\}$$

a) Write down two sequences in W.

b) Write down a sequences that is NOT in W (if possible).

c) Show that W is a subspace of $\mathcal{S}$.