# Exam 2 Fall 23 2:03

1) a) vector addition and scalar multiplication

b) $4 \times 4$ matrices with real entries

c) $A$ is invertible

d) i) possible

ii) possible

iii) not possible

iv) possible

2) a)
$$\begin{bmatrix} \frac{1}{8} & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

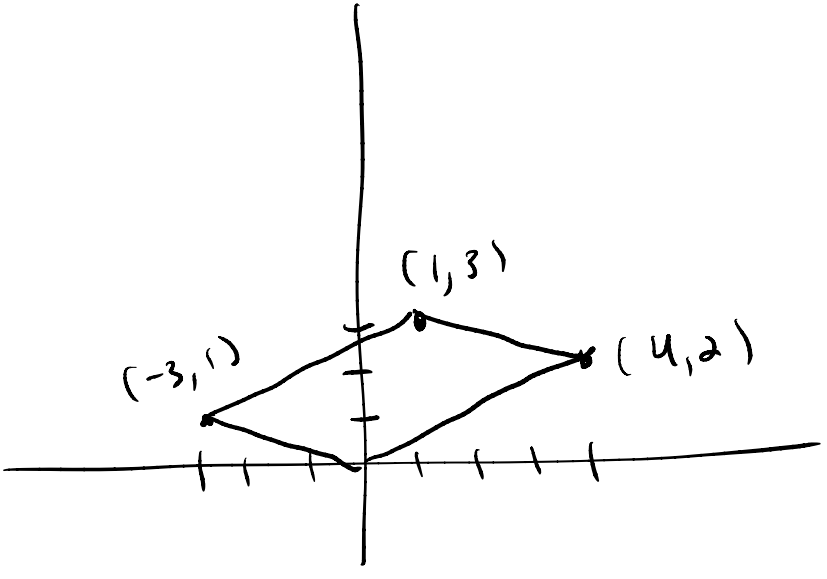
b)
$$\begin{bmatrix} \cos(\frac{2\pi}{3}) & -\sin(-\frac{2\pi}{3}) & 0 \\ \sin(-2\pi/3) & \cos(-2\pi/3) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

c)
$$\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix}$$

d) $B \cdot C \cdot A$

3) a)




$$\text{Area} = |\det\left(\begin{bmatrix} -3 & 4 \\ 1 & 2 \end{bmatrix}\right)|$$

$$= |-3 \cdot 2 - 4 \cdot 1|$$

$$= 10$$

b) $\|\vec{v}\|_2 = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$$\vec{v} = \begin{bmatrix} \sqrt{2}\cos(\theta) \\ \sqrt{2}\sin(\theta) \end{bmatrix}$$

$\theta = \arctan\left(\frac{-1}{1}\right)$

$= -\pi/4$

$= -45^\circ$

$$\vec{\omega} = \begin{bmatrix} 5\cos(\theta + 30) \\ 5\sin(\theta + 30) \end{bmatrix}$$

$$\vec{\omega} = \begin{bmatrix} 5\cos(-15^\circ) \\ 5\sin(-15^\circ) \end{bmatrix} \approx \begin{bmatrix} 4.8296 \\ -1.2941 \end{bmatrix}$$

$$-\text{or}- \quad \vec{w} = \begin{bmatrix} 5\cos(75^\circ) \\ 5\sin(75^\circ) \end{bmatrix} \approx \begin{bmatrix} 1.2941 \\ -4.8296 \end{bmatrix}$$

$$-\text{or}-$$

$$\cos(30^\circ) = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\|_2 \|\vec{w}\|} = \frac{\begin{bmatrix} 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}}{\sqrt{2} \cdot 5}$$

$$\frac{\sqrt{3}}{2} = \frac{x-y}{5\sqrt{2}}$$

$$\frac{5\sqrt{3}}{\sqrt{2}} + y = x$$

$$x^2 + y^2 = 25$$

$$\left(\frac{5\sqrt{3}}{\sqrt{2}} + y\right)^2 + y^2 = 25$$

$$\frac{75}{2} + 5\sqrt{6}\,y + 2y^2 = 25$$

$$2y^2 + 5\sqrt{6}\,y + \frac{25}{2} = 0$$

$$y = \frac{-5\sqrt{6} \pm \sqrt{150 - 100}}{4}$$

$$y = \frac{-5\sqrt{6} \pm 5\sqrt{2}}{4}$$

$$x = \frac{5\sqrt{3}}{\sqrt{2}} + y$$

$$x = \frac{5\sqrt{3}}{\sqrt{2}} + \frac{-5\sqrt{6} \pm 5\sqrt{2}}{4}$$

4) a) $(0, 0, 0, \ldots)$

$$\sum_{n=1}^{\infty} 0 = 0$$

$(1, -1, 0, 0, 0, \ldots)$

$$1 + (-1) + \sum_{n=3}^{\infty} 0 = 0$$

b) $(1, 1, 1, \ldots)$

$$\sum_{n=1}^{\infty} 1 \quad \text{diverges}$$

c) Suppose $(a_n), (b_n) \in W$, $k \in \mathbb{R}$

$$\sum_{n=1}^{\infty} a_n = 0 = \sum_{n=1}^{\infty} b_n$$

$(a_n) + (b_n) = (a_n + b_n)$

$$\sum_{n=1}^{\infty} (a_n + b_n) = \sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$$

$$= 0 + 0$$

$$= 0$$

$k \cdot (a_n) = (k \cdot a_n)$

$$\sum_{n=1}^{\infty} k \cdot a_n = k \cdot \sum_{n=1}^{\infty} a_n$$

$$= k \cdot 0$$

$$= 0$$

So $W$ is a subspace