Name:

Math 227 Exam 2

March 18, 2013

Directions: WRITE YOUR NAME ON THIS TEST! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. DO NOT convert irrational numbers such as $\sqrt{3}$ or π into decimal approximations; just leave them as they are.

1) (2 points each) True/False. If the sentence is false, correct the error. No justification is necessary.

a) The dimension of $Col(A)$ is equal to $Rank(A)$ for all matrices A.

b) If a vector space V has dimension n, then V can have a basis with less than n vectors in it.

c) Every linear transformation between finite-dimensional vector spaces may be written as a matrix.

d) If an $n \times n$ matrix A is invertible, then $\det(A) = \dfrac{1}{\det(A^{-1})}$.

e) The vector space of continuous functions from $\mathbb{R}$ to $\mathbb{R}$ is finite dimensional.

2) Find the matrix of the linear transformations on $\mathbb{R}^3$ that, in homogeneous coordinates

a) (4 points) scales a 2-vector down by a factor of 2

b) (5 points) shifts a 2-vector up 7 units and left 6 units

c) (6 points) rotates a 2-vector by $\pi/4$ radians counterclockwise

d) (8 points) first scales a vector down by a factor of 2, then shifts the vector up 7 units and left 6 units, then rotates the vector by $\pi/4$ radians counterclockwise.

3) Let $A = \begin{bmatrix} 0 & 4 & -2 & 4 \\ -4 & -10 & 1 & -6 \\ 7 & -4 & 9 & -11 \end{bmatrix}$

a) (7 points) Define the nullspace $Nul(A)$ and the range space $Ran(A)$.

b) (10 points) Calculate $Ran(A)$ and find a basis for it.

c) (6 points) State the Rank Theorem and use it to find the dimension of $Nul(A)$ without doing any matrix operations.

4) Let $A = \begin{bmatrix} 3 & 2 & 0 \\ 0 & 1 & 4 \\ 0 & 6 & -1 \end{bmatrix}$.

a) (4 points) Find the determinant of A. Is A invertible?

b) (9 points) If A is invertible, calculate the inverse. If A is not invertible, find a basis for $Nul(A)$.

c) (6 points) Find the matrix of A with respect to the basis $\{b_1, b_2, b_3\}$ where

$$b_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \; b_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \; \text{and } b_3 = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

5) Let $w = \begin{bmatrix} 4 \\ -6 \\ 17 \end{bmatrix}$ and define $T : \mathbb{R}^3 \to \mathbb{R}$ by $T(v) = v \cdot w$, the dot product of v with w.

a) (7 points) Check that T is a linear transformation.

b) (8 points) Let $W \subseteq \mathbb{R}^3$ be the subset of all vectors v satisfying $3v \cdot w - 7w \cdot v = 0$. Check that W is a subspace of $\mathbb{R}^3$.

c) (10 points) Find a linear transformation $S : \mathbb{R}^3 \to \mathbb{R}^3$ with $Ran(S) = W$ where W is the subspace from part b). You don't have to check that S is linear, just find it.

BONUS: (10 points) For all matrices P in $M_n(\mathbb{R})$ satisfying $P \cdot P = P$, if $T = I_n - 2P$, show that T is always invertible.