Math 227 Exam 2 Winter 13 Solutions

1)

2) a) $\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & -6 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} \sqrt{2}/4 & -\sqrt{2}/4 & -13\sqrt{2}/2 \\ \sqrt{2}/4 & \sqrt{2}/4 & \sqrt{2}/2 \\ 0 & 0 & 1 \end{bmatrix}$

3)

a) $Nul(A) = \{v \in \mathbb{R}^4 : \ Av = 0\}$, $Ran(A) = \{w \in \mathbb{R}^3 : \ \exists \ v, \ Av = w\}$

b) basis $\begin{bmatrix} 0 \\ -4 \\ 7 \end{bmatrix}, \begin{bmatrix} 4 \\ -10 \\ -4 \end{bmatrix}$; range $x_1 \begin{bmatrix} 0 \\ -4 \\ 7 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ -10 \\ -4 \end{bmatrix}$ for all scalars x_1 and x_2.

c) If $A : \mathbb{R}^n \to \mathbb{R}^m$, $Rank(A) + \dim(Nul(A)) = n$, so $\dim(Nul(A)) = 2$.

4) a) $-75 \neq 0$, so invertible

b) $A^{-1} = \begin{bmatrix} 1/3 & -2/75 & -8/75 \\ 0 & 1/25 & 12/75 \\ 0 & 6/25 & -1/25 \end{bmatrix}$.

c) $\begin{bmatrix} 3/2 & 0 & 0 \\ 0 & 1/2 & 2 \\ 0 & 3 & 0 \end{bmatrix}$

5) a) Do it.

b) $W = Nul(T)$ and so is a subspace.

c) $S((x, y, z)) = x \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} + y \begin{bmatrix} 0 \\ 17 \\ 6 \end{bmatrix}$