Name:

Math 227 Exam 2

March 12, 2015

Directions: WRITE YOUR NAME ON THIS TEST! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. Decimal approximations, accurate to four decimal places, are acceptable.

1) Find the matrix of the linear transformations on $\mathbb{R}^3$ that, in homogeneous coordinates,

a) (3 points) scales the x-coordinate of a 2-vector down by 3 and the y-coordinate up by 7.

b) (5 points) shifts a 2-vector up 13 units and left 8 units

c) (6 points) rotates a 2-vector by $2\pi/3$ radians counterclockwise

d) (6 points) scales the x-coordinate of a 2-vector down by 3 and the y-coordinate up by 7, then rotates the vector by $2\pi/3$ radians counterclockwise, and finally shifts the vector up 13 units and left 8 units.

2) a) (12 points) Let $W = \{(x, y, z, w) \in \mathbb{R}^4 \mid x + w = y + z\}$. Show that W is a subspace of $\mathbb{R}^4$.

b) (10 points) Let $\mathcal{S} = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_2(\mathbb{R}) \mid ad = 0 \right\}$. Show that $\mathcal{S}$ is NOT a subspace of $M_2(\mathbb{R})$.

3) Given the signals $x_k = 1$, $w_k = (1/2)^k$, and $w_k = k$ and the homogeneous linear difference equation

$$2y_{k+3} - 3y_{k+2} + y_k = 0, \tag{1}$$

a) (12 points) check that $(x_k)_{k\in\mathbb{Z}}$, $(w_k)_{k\in\mathbb{Z}}$, and $(z_k)_{k\in\mathbb{Z}}$ all satisfy equation (1);

b) (8 points) determine the Casorati matrix associated to the signals $(x_k)_{k\in\mathbb{Z}}$, $(w_k)_{k\in\mathbb{Z}}$, and $(z_k)_{k\in\mathbb{Z}}$.

c) (10 points) Is $\{(x_k)_{k\in\mathbb{Z}}, (w_k)_{k\in\mathbb{Z}}, (z_k)_{k\in\mathbb{Z}}\}$ a basis for the subspace of all signals satisfying equation (1)? Justify your assertion.

4) a) (6 points) If

$$A = \begin{bmatrix} 2 & 5 & -1 & 0 \\ 3 & 6 & -1 & 1 \\ 0 & -12 & 4 & 8 \\ -1 & 14 & -5 & -11 \end{bmatrix},$$

find two distinct nonzero vectors in the nullspace (kernel) of A and two distinct nonzero vectors in the column space (range) of A.

b) (12 points) Define $T : M_2(\mathbb{R}) \to M_2(\mathbb{R})$,

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} \frac{a+d}{2} & 0 \\ 0 & \frac{a+d}{2} \end{bmatrix}.$$

Determine $\ker(T)$, with calculations to support your answer.