Name:

Math 227 Exam 2

March 22, 2018

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Wolfram Alpha or a similar program may be used for all computational problems.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.

1) Find the matrix of the linear transformations on $\mathbb{R}^3$ that, in homogeneous coordinates,

a) (4 points) scales a 2-vector up by a factor of 17,

b) (8 points) shifts a 2-vector down 4 units and right 11 units,

c) (6 points) rotates a 2-vector by $\pi/6$ radians clockwise,

d) (6 points) first shifts a 2-vector down 4 units and right 11 units, then rotates a 2-vector by $\pi/6$ radians clockwise, and finally scales a 2-vector up by a factor of 17.

2) Let T be the linear transformation from $\mathbb{R}^2 \to \mathbb{R}^3$ given by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} y - x \\ y + 2x \\ x + 4y \end{bmatrix}.$$

a) (7 points) Compute a matrix A that implements T.

b) (7 points) Determine whether $A^t A$ is invertible or not.

c) (7 points) Calculate the orthogonal projection onto $\mathrm{Ran}(T)$.

3) Given the points $(0,6)$, $(-1,2)$, $(-5,7)$ and $(3,4)$ in $\mathbb{R}^2$, find the best-fit line to the points by

a) (12 points) Finding a system of linear equations that represents a "solution" to the problem,

b) (6 points) Writing the problem as a matrix equation $Ax = b$,

c) (6 points) Finding the system $A^t Ax = A^t b$, computing both $A^t A$ and $A^t b$,

d) (7 points) Solving the system in c) and producing the polynomial.

4) a) (13 points) Let $W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_2(\mathbb{R}) \mid a + d = 0 \right\}$. Show that W is a subspace of $M_2(\mathbb{R})$.

b) (11 points) Let $S = \{(x, y) \in \mathbb{R}^2 \mid y \geq 0\}$. Show that S is NOT a subspace of $\mathbb{R}^2$.

BONUS: (10 points) Let $\mathbb{P}$ be the vector space of all polynomials. Define $T : \mathbb{P} \to \mathbb{P}$ by

$$T(p(x)) = p'(x)$$

Determine $\ker(T)$, with proof.