Name:

Math 227 Exam 2

March 12, 2020

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.

1) Let V, W be a vector spaces. Let $T : V \to W$ be a linear function.

a) (4 points) What are the two operations on V, i.e., what makes a vector space?

b) (3 points) Let $V = \mathcal{F}(\mathbb{R})$. What are the vectors?

c) (2 points) Which vector space is $\mathrm{Ran}(T)$ a subspace of?

d) (2 points) Which vector space is $\ker(T)$ a subspace of?

2) Find a single 3×3 matrix that, in homogeneous coordinates,

a) (6 points) scales a 2-vector up by a factor of 17,

b) (7 points) shifts a 2-vector down 6 units and right 11 units,

c) (8 points) rotates a 2-vector by $\pi/4$ radians clockwise,

d) (6 points) does a)-c) in order, starting with a).

3) Let $T : \mathbb{R}^5 \to \mathbb{R}^2$,

$$T\left(\begin{bmatrix} w \\ u \\ x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 9w + 4x - z \\ -12y + 13z + u \end{bmatrix}.$$

a) (8 points) Show that T is linear.

b) (8 points) Find three nonzero, nonparallel vectors in $\mathrm{Ran}(T)$.

c) (8 points) Find a nonzero vector in $\ker(T)$.

4) (20 points) Let

$$W = \{(x, y, z) \in \mathbb{R}^3 \mid 42z = 19x\}.$$

Show that W is a subspace of $\mathbb{R}^3$.

5) (18 points) Let

$$S = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_2(\mathbb{R}) \mid (a-b)(c-d) = 0 \right\}.$$

Show that S is NOT a subspace of $M_2(\mathbb{R})$.