

Name:

Math 227 Exam 2

March 17, 2022

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.

1) Let V, W be vector spaces.

a) What are the two operations on V , i.e., what makes a vector space?

b) Let $V = \mathbb{P}[\mathbb{R}]$. What are the vectors?

c) If an $n \times n$ matrix is NOT invertible, what can you say about the determinant?

- 2) Find a single 3×3 matrix that, in homogeneous coordinates,
- a) rotates a 2-vector by $4\pi/3$ radians clockwise,
 - b) shifts a 2-vector up 12 units and right 9 units,
 - c) scales the x -coordinate of a 2-vector down by a factor of 4 and scales the y -coordinate up by a factor of 10,
 - d) does a)-c) in order, starting with a).

3) Let

$$A = \begin{bmatrix} -2 & 5 \\ 7 & 6 \end{bmatrix}$$

a) Is A invertible? Why or why not?

b) Find the area of the parallelogram with vertices $(0, 0)$, $(-2, 5)$, $(7, 6)$, and $(5, 11)$. Be sure to draw a picture!

4) Let $W \subseteq M_2(\mathbb{R})$

$$W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a + 4c = -b + 3d \right\}$$

- a) Write down three matrices in W .
- b) Write down a matrix that is NOT in W (if possible).
- c) Show that W is a subspace of $M_2(\mathbb{R})$.

5) Let

$$S = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \mid xy - z = 0 \right\} \subseteq \mathbb{R}^3$$

Show that S is NOT a subspace of \mathbb{R}^3 .