Exam 2 W '23

1) a) vector addition and scalar multiplication

b) 4×4 matrices with real entries

c) A is not invertible

d) W

e) V

2) a) $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -15 \\ 0 & 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1/7 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} \cos(11\pi/6) & -\sin(11\pi/6) & 0 \\ \sin(11\pi/6) & \cos(11\pi/6) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} \sqrt{3}/2 & 1/2 & 0 \\ -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

d) $B \cdot A \cdot C$

3) a) $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ = 1st column

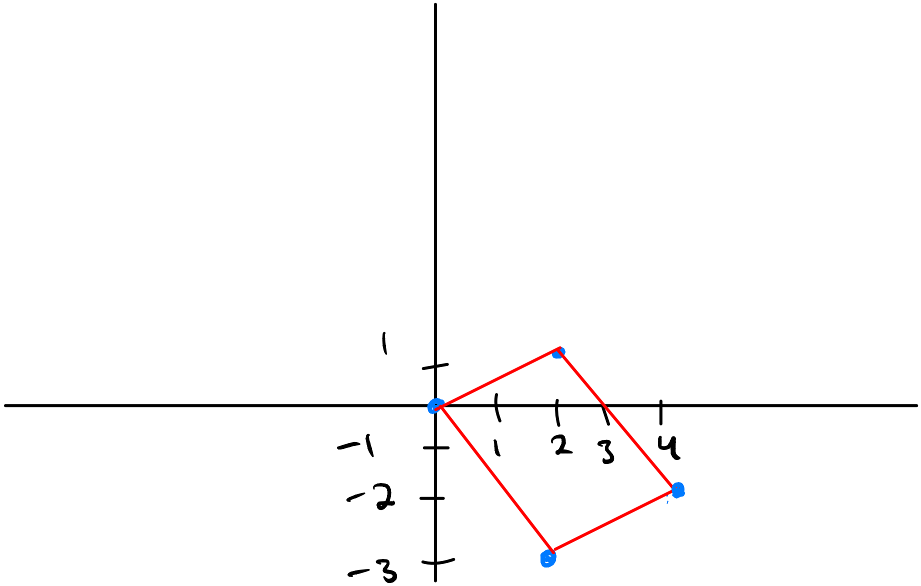
$T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ = 2nd column

$$A = \begin{bmatrix} 2 & 2 \\ 1 & -3 \end{bmatrix}$$

b) $\det(A) = 2 \cdot (-3) - 2 \cdot 1 = -8 \neq 0$

so A is invertible

c)



Area = $|\det(A)| = 8$

4) a) $(0, 0, 0, 0, \text{---})$

$(0, 1, 0, 0, \text{---})$

b) $(1, 1, 1, 1, \text{---})$

c) If $a_1^2 + a_3^2 = 0$, then

$a_1 = a_3 = 0$,

so $W = \{ (a_n)_{n=1}^{\infty} \mid a_1 = a_3 = 0 \}$.

By a), W is nonempty.

Addition:

Let $(a_n)_{n=1}^{\infty}$, $(b_n)_{n=1}^{\infty} \in W$.

Then $a_1 = a_3 = b_1 = b_3 = 0$.

$$a_1 + b_1 = 0 + 0 = 0 = a_3 + b_3$$

so $(a_n)_{n=1}^{\infty} + (b_n)_{n=1}^{\infty}$

$= (a_n + b_n)_{n=1}^{\infty} \in W$

Scalar Multiplication: let $c \in \mathbb{R}$.

Then $c \cdot a_1 = c \cdot 0 = 0 = c \cdot a_3$

So $c \cdot (a_n)_{n=1}^{\infty} = (c \cdot a_n)_{n=1}^{\infty} \in W$

So W is a subspace!

5) a) $\frac{A + A^t}{2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

b) $\frac{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}^t}{2}$

$= \frac{1}{2} \cdot \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

c) $\frac{\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} a & c \\ b & d \end{bmatrix}}{2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= \frac{1}{2} \begin{bmatrix} 2a & b+c \\ b+c & 2d \end{bmatrix}$

So $a = 0$, $d = 0$,

$b + c = 0 \Rightarrow b = -c$

$$\ker(T) = \left\{ \begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix} \;\middle|\; a \in \mathbb{R} \right\}$$