

Name:

Math 227 Exam 3

December 2, 2021

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.

1) Let

$$A = \begin{bmatrix} -29 & -18 \\ 54 & 34 \end{bmatrix}.$$

a) (11 points) Compute all eigenvalues of A BY HAND.

b) (8 points) Check that $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ are eigenvectors corresponding to the eigenvalues you found in a) BY HAND.

2) Given the points $(1, -4)$, $(-1, 2)$, $(2, 3)$ and $(-4, 7)$ in \mathbb{R}^2 , find the best-fit **LINE** to the points by

a) (8 points) Finding a system of linear equations that represents a “solution” to the problem,

b) (8 points) Writing the problem as a matrix equation $Ax = b$,

c) (6 points) Finding the system $A^tAx = A^tb$, computing both A^tA and A^tb ,

d) (6 points) Solving the system in c) and producing the polynomial.

3) Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$,

$$T \left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \right) = \begin{bmatrix} 12x - 4z + w \\ 4x + 5y + 6z - 16w \\ 8z - 6w \end{bmatrix}.$$

- a) (15 points) Determine a matrix representation A for T .
- b) (5 points) Find four nonzero, nonparallel vectors in $\text{Ran}(T)$. You do not have to show your work.
- c) (5 points) Find a nonzero vector in $\ker(T)$.

4) Given the simplified link diagram between webpages P_1 , P_2 , and P_3 described by

- P_1 links to P_2
- P_2 doesn't link to anything,
- P_3 links to P_1 ,

a) (5 points) Construct the link matrix A .

b) (6 points) Find the normalized matrix B .

c) (11 points) Calculate the PageRank matrix C , using $d = .85 = 17/20$.

4) (continued) d) (2 points) What number is the matrix C guaranteed to have as an eigenvalue?

e) (4 points) If an associated eigenvector v to the eigenvalue from d) is

$$\begin{bmatrix} 740 \\ 1029 \\ 400 \end{bmatrix}$$

find the PageRank of P_2 .