Name:

Math 227 Exam 3

December 5, 2018

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Wolfram Alpha or a similar program may be used for all computational problems.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.

1) Let

$$A = \begin{bmatrix} 7 & -21 \\ -14 & 42 \end{bmatrix}.$$

a) (13 points) Compute all eigenvalues of A BY HAND.

b) (4 points) Find an associated eigenvector for each eigenvalue from a).

c) (8 points) Check that each actually vector from b) actually is an eigenvector BY HAND.

2) Given the simplified link diagram between webpages P_1, P_2, and P_3 described by

- P_1 links to P_2
- P_2 links to P_1
- P_3 links to P_1 and P_2.

a) (5 points) Construct the link matrix A.

b) (6 points) Find the normalized matrix B.

c) (11 points) Calculate the PageRank matrix C, using $d = .85 = 17/20$.

2) (continued) d) (2 points) What number is the matrix C guaranteed to have as an eigenvalue?

e) (4 points) If an associated eigenvector v to the eigenvalue from d) is

$$\begin{bmatrix} 19/2 \\ 19/2 \\ 1 \end{bmatrix}$$

find the PageRank of P_3.

3) Let $T : \mathbb{R}^5 \to \mathbb{R}^3$,

$$T\left(\begin{bmatrix} x \\ y \\ z \\ w \\ u \end{bmatrix}\right) = \begin{bmatrix} 2x + 3z + w + u \\ 10x + 2y + 14z + 4w + 6u \\ 11x - 5y + 19z + 8w + 3u \end{bmatrix}.$$

a) (10 points) Determine a matrix representation A for T.

b) (6 points) Show that the dimension of $\mathrm{Ran}(T)$ is greater than 1.

c) (6 points) Show that the dimension of $\mathrm{Ran}(T)$ is less than 3, i.e., show that there is a vector NOT in $\mathrm{Ran}(T)$.

d) (4 points) Find a basis for $\mathrm{Ran}(T)$.

4) a) (8 points) Show that if $T : V \to V$ is linear and x, y are eigenvectors for T associated to the eigenvalue λ, then $x + y$ is also an eigenvector if $x \neq -y$.

b) (13 points) Let $\{v_1, v_2, v_3\}$ be a basis for a vector space V and let $S : V \to V$ be linear and invertible. Show that

$$\{Sv_1, Sv_2, Sv_3\}$$

is also a basis for V.

BONUS: (10 points) Let $\mathcal{F}(\mathbb{R})$ be the space of all functions from $\mathbb{R}$ to $\mathbb{R}$ and let $f, g \in \mathcal{F}(\mathbb{R})$. Define $T_g : \mathcal{F}(\mathbb{R}) \to \mathcal{F}(\mathbb{R})$ by

$$T_g(f(x)) = f(g(x)).$$

Show that, for ALL choices of g, $\lambda = 1$ is an eigenvalue of T_g.