Exam 3 F 18

1)

a)

$$0 = \det\left\{\begin{bmatrix} 7 & -21 \\ -14 & 42 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right\}$$

$$= \det\begin{pmatrix} 7-\lambda & -21 \\ -14 & 42-\lambda \end{pmatrix}$$

$$= \lambda^2 - 49\lambda + 21 \cdot 14 - 21 \cdot 14$$

$$= \lambda^2 - 49\lambda = \lambda(\lambda - 49)$$

$$\lambda = 0, \quad \lambda = 49$$

b) $\overset{\lambda=0}{\begin{bmatrix} 3 \\ 1 \end{bmatrix}}, \overset{\lambda=49}{\begin{bmatrix} 1 \\ -2 \end{bmatrix}}$

c) $$\begin{bmatrix} 7 & -21 \\ -14 & 42 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$= 0 \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 7 & -21 \\ -14 & 42 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 49 \\ -98 \end{bmatrix}$$

$$= 49 \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

2)

$$\begin{tikzcd} & P2 \arrow[dl, shift left] & \\ P1 \arrow[ur, shift left] & & P3 \arrow[ul] \arrow[ll] \end{tikzcd}$$

a) $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

b) $B = \begin{bmatrix} 0 & 1 & 1/2 \\ 1 & 0 & 1/2 \\ 0 & 0 & 0 \end{bmatrix}$

c) $C = \frac{17}{20} \begin{bmatrix} 0 & 1 & 1/2 \\ 1 & 0 & 1/2 \\ 0 & 0 & 0 \end{bmatrix} + \frac{\frac{3}{20}}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1/20 & 9/10 & 19/40 \\ 9/10 & 1/20 & 19/40 \\ 1/20 & 1/20 & 1/20 \end{bmatrix}$$

d) $\lambda = 1$

e) $\frac{1}{19+1}\begin{bmatrix} 19/2 \\ 19/2 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 19/40 \\ 19/40 \\ 1/20 \end{bmatrix}$

$p_3 = 1/20$

3) a) $T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 10 \\ 11 \end{bmatrix}$

$T\left(\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 2 \\ -5 \end{bmatrix}$

$T\left(\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 14 \\ 19 \end{bmatrix}$

$T\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 4 \\ 8 \end{bmatrix}$

$T\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix}$

$$A = \begin{bmatrix} 2 & 0 & 3 & 1 & 1 \\ 10 & 2 & 14 & 4 & 6 \\ 11 & -5 & 19 & 8 & 3 \end{bmatrix}$$

b) $T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}2\\10\\11\end{bmatrix}$ and

$T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}0\\2\\-5\end{bmatrix}$ are

in $\mathrm{Ran}(T)$ and are not

scalar multiples, so

$\dim(\mathrm{Ran}(T)) > 1$.

c) $\mathrm{RREF}\begin{bmatrix} A & \begin{matrix}1\\0\\0\end{matrix}\end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 3/2 & 1/2 & 1/2 & 0 \\ 0 & 1 & -1/2 & -1/2 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

so $\begin{bmatrix}1\\0\\0\end{bmatrix} \notin \mathrm{Ran}(A) = \mathrm{Ran}(T)$

d) $\begin{bmatrix} 3 \\ 10 \\ 11 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -5 \end{bmatrix}$

4) a) $T(x+y)$

$$= T(x) + T(y)$$

$$= \lambda x + \lambda y$$

$$= \lambda (x+y)$$

b) Suppose $\exists\ \alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$

$$\alpha_1 \underbrace{Sv_1 + \alpha_2 Sv_2}_{} + \alpha_3 Sv_3 = 0_V$$

$$= S\alpha_1 v_1 + S\alpha_2 v_2 + S\alpha_3 v_3$$

$$= S(\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3)$$

Applying S^{-1}, $\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 = 0_V$

$\Rightarrow \alpha_1 = \alpha_2 = \alpha_3 = 0$ since

$\{v_1, v_2, v_3\}$ is a basis.

Therefore, $\{Sv_1, Sv_2, Sv_3\}$ is

linearly independent, and since

$\dim(V_3) = 3$, we have a basis.