Name:

Math 227 Exam 3

December 5, 2019

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Wolfram Alpha or a similar program may be used for all computational problems.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.

1) Let

$$A = \begin{bmatrix} 14 & -9 \\ 18 & -13 \end{bmatrix}.$$

a) (11 points) Compute all eigenvalues of A BY HAND.

b) (4 points) Find an associated eigenvector for each eigenvalue from a).

c) (8 points) Check that each vector from b) actually is an eigenvector BY HAND.

2) Given the points $(0, 2)$, $(-3, 4)$, $(5, 8)$ and $(9, -16)$ in $\mathbb{R}^2$, find the best-fit quadratic to the points by

a) (10 points) Finding a system of linear equations that represents a "solution" to the problem,

b) (9 points) Writing the problem as a matrix equation $Ax = b$,

c) (4 points) Finding the system $A^t Ax = A^t b$, computing both $A^t A$ and $A^t b$,

d) (4 points) Solving the system in c) and producing the polynomial.

3) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$,

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 8x - y + 5z \\ -42x + 13y - 7z \end{bmatrix}.$$

a) (9 points) Determine a matrix representation A for T.

b) (5 points) Find four nonzero, nonparallel vectors in $\mathrm{Ran}(T)$.

c) (3 points) Find a nonzero vector in $\ker(T)$.

d) (8 points) Compute the orthogonal projection onto $\ker(T)$. You don't have to show it is an orthogonal projection, just find it.

4) (10 points) Let $V = \mathcal{F}(\mathbb{R})$, the functions from $\mathbb{R}$ to $\mathbb{R}$. Define $T : V \to V$ by

$$T(f)(x) = x^2 f(x),$$

Show that T is linear.

5) (15 points) Let P and Q be orthogonal projections from $\mathbb{R}^n$ to $\mathbb{R}^n$. Show that if $PQ = 0$, then $P + Q$ is also an orthogonal projection.