Name:

Math 227 Exam 3

November 30, 2023

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.

1) Let

$$A = \begin{bmatrix} -68 & 46 \\ -138 & 93 \end{bmatrix}.$$

a) Compute all eigenvalues of A BY HAND.

b) What is the one vector in $\mathbb{R}^2$ that has no possibility of being an eigenvector for A?

c) If $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$ is orthogonal to one of the eigenvectors of A, find an eigenvector of A.

2) Given the points $(2,4)$, $(-3,3)$, $(4,0)$, and $(1,2)$, in $\mathbb{R}^2$, find the best-fit **LINE** to the points by

a) Finding a system of linear equations that represents a "solution" to the problem,

b) Writing the problem as a matrix equation $A\vec{x} = \vec{b}$,

c) Finding the system $A^t A\vec{x} = A^t\vec{b}$, computing both $A^t A$ and $A^t\vec{b}$,

d) Solving the system in c) and producing the polynomial.

3) Given the simplified link diagram between webpages P_1, P_2, and P_3 described by

- P_1 links to P_3
- P_2 doesn't link to anything,
- P_3 links to P_1 and P_2,

a) Construct the link matrix A.

b) Find the normalized matrix B.

c) Calculate the PageRank matrix C, using $d = .85 = 17/20$.

3) (continued) d) What number is the matrix C guaranteed to have as an eigenvalue?

e) If an associated eigenvector $\vec{v}$ to the eigenvalue from d) is

$$\begin{bmatrix} 57 \\ 57 \\ 74 \end{bmatrix}$$

find the PageRank of P_3.

4) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$,

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + 6y - 3z \\ -7x - 42y + 21z \end{bmatrix}.$$

a) Determine a matrix representation A for T.

b) Find two vectors in $\ker(T)$ that are not multiples of eachother. Geometrically, what does this tell you $\ker(T)$ must be?

c) Find the closest vector in $\mathrm{Ran}(T)$ to $\begin{bmatrix} 7 \\ 2 \end{bmatrix}$.