Name:

Math 227 Exam 3

April 10, 2013

Directions: WRITE YOUR NAME ON THIS TEST! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. DO NOT convert irrational numbers such as $\sqrt{3}$ or π into decimal approximations; just leave them as they are.

1) (2 points each) True/False. If the sentence is false, correct the error. No justification is necessary.

a) Every $n \times n$ matrix is diagonalizable.

b) An orthogonal matrix can have zero as an eigenvalue.

c) If $Av = \lambda v$ and $v \neq 0$, then v is an eigenvector of A.

d) If A is in $M_n(\mathbb{R})$ and $\|Av\|_2 \leq c\|v\|_2$ for all vectors v in $\mathbb{R}^n$, then $\|A\| = c$.

e) The PageRank of a webpage is a real number in between 0 and 1.

2) Given the signals $x_k = 3^k$ and $z_k = k(3^k)$ and the homogeneous linear difference equation

$$y_{k+2} - 6y_{k+1} + 9y_k = 0, \qquad (1)$$

a) (15 points) check that (x_k) and (z_k) all satisfy equation (1);

b) (5 points) determine the Casorati matrix associated to the signals (x_k) and (z_k).

c) (10 points) Is $\{(x_k), (z_k)\}$ a basis for the subspace of all signals satisfying equation (1)? Justify your assertion.

3) (20 points) Given the simplified link diagram between webpages P_1, P_2 and P_3 described by

- P_1 doesn't link to anything
- P_2 links to P_1 and P_3
- P_3 links to P_1

find the PageRank of P_2, using $d = .85$. SHOW YOUR STEPS.

4) Let $A = \begin{bmatrix} 5 & -3 \\ -3 & 5 \end{bmatrix}$.

a) (6 points) Find the eigenvalues of A and an eigenvector associated to each eigenvalue.

b) (7 points) Calculate the norm of A. Show the procedure of arriving at your answer.

c) (12 points) Determine the polar decomposition of A.

5) (15 points) Recall that if $v = \langle v_1, v_2, v_3 \rangle$ and $w = \langle w_1, w_2, w_3 \rangle$ are vectors in $\mathbb{R}^3$, then the cross product $v \times w$ is given by

$$v \times w = \langle v_2 w_3 - w_2 v_3, v_3 w_1 - w_3 v_1, v_1 w_2 - w_1 v_2 \rangle.$$

Let $v = \langle 1, 2, -1 \rangle$ and define the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ by $T(w) = v \times w$. Find all real eigenvalues of T.

BONUS: (10 points) For all $n \times n$ matrices A, show that if λ is an eigenvalue for A then λ is also an eigenvalue for A^t.