Name:

Math 227 Exam 3

April 9, 2015

Directions: WRITE YOUR NAME ON THIS TEST! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. Decimal approximations, accurate to four decimal places, are acceptable.

1) Given the simplified link diagram between webpages P_1, P_2, P_3 and P_4 described by

- P_1 links to P_2 and P_4
- P_2 links to P_3 and P_1
- P_3 doesn't link to anything
- P_4 links to P_1, P_2, and P_3,

a) (4 points) Construct the link matrix A.

b) (6 points) Find the normalized matrix B.

c) (11 points) Calculate the PageRank matrix C, using $d = .85 = 17/20$.

d) (6 points) Find the associated eigenvector v with all positive entries whose 1-norm is equal to one and find the PageRank of P_4.

2) a) (8 points) Let

$$C = \begin{bmatrix} 13 & -4 & 1 \\ 14 & -2 & -2 \\ -12 & 6 & -4 \end{bmatrix}.$$

Find all eigenvalues of C, and for each eigenvalue, find an associated eigenvector with 2-norm equal to 3.

b) (10 points) Let $A, B \in M_n(\mathbb{R})$ and suppose $AB = BA$. Show that if x is an eigenvector for A, then so is Bx (provided $Bx \neq 0$).

3) Given the points $(-1, 2)$, $(2, -1)$, $(5, 1)$, and $(7, 3)$, find the best-fit quadratic to the points by

a) (7 points) Finding a system of linear equations that represents a "solution" to the problem,

b) (8 points) Writing the problem as a matrix equation $Ax = b$,

c) (6 points) Finding the system $A^t Ax = A^t b$, computing both $A^t A$ and $A^t b$,

d) (6 points) Solving the system in c) and producing the polynomial.

4) a) (8 points) Let

$$v = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 2 \end{bmatrix} \text{ and } w = \begin{bmatrix} -4 \\ 3 \\ 6 \\ 0 \end{bmatrix}.$$

Find the orthogonal projection $P \in M_4(\mathbb{R})$ onto the span of the vector v, then compute Pw.

b) (10 points) Let $P \in M_n(\mathbb{R})$ be an orthogonal projection. Show that $\det(P)$ is either zero or one.