Name:

Math 227 Exam 3

April 12, 2018

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Wolfram Alpha or a similar program may be used for all computational problems.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.

1) Let

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -2 \end{bmatrix}.$$

a) (13 points) Compute all eigenvalues of A BY HAND.

b) (4 points) Find an associated eigenvector for each eigenvalue from a).

c) (8 points) Check that each actually vector from b) actually is an eigenvector BY HAND.

2) Given the simplified link diagram between webpages P_1, P_2, and P_3 described by

- P_1 links to P_2 and P_3
- P_2 doesn't link to anything.
- P_3 links to P_2.

a) (5 points) Construct the link matrix A.

b) (6 points) Find the normalized matrix B.

c) (11 points) Calculate the PageRank matrix C, using $d = .85 = 17/20$.

d) (6 points) Find the associated eigenvector v with all positive entries whose 1-norm is equal to one and find the PageRank of P_1.

3) Let $T : \mathbb{R}^4 \to \mathbb{R}^2$,

$$T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} 9x - 2y + w \\ -2x - y - 12z \end{bmatrix}.$$

a) (9 points) Determine a matrix representation for T.

b) (12 points) Find a basis for $\ker(T)$. What is the dimension of $\ker(T)$?

c) (3 points) Compute an orthonormal basis for $\ker(T)$. A correct answer here will be sufficient for credit in the first part of b).

4) a) (8 points) Show that if $T : V \to V$ is linear and x is an eigenvector for T associated to the eigenvalue λ, then cx is also an eigenvector for any $c \neq 0$.

b) (15 points) Let $\{v_1, v_2, v_3\}$ be a basis for a vector space V. Show that

$$\{v_1 + v_2, v_2 + v_3, v_3 + v_1\}$$

is also a basis for V.

BONUS: (10 points) Let S be the space of all sequences of real numbers and define $P : S \to S$,

$$P((a_n)_{n=1}^{\infty}) = (a_1, 0, a_3, 0, a_5, 0, \dots)$$

Find all eigenvalues of P.