Name:

Math 227 Exam 3

April 14, 2022

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.

1) Let

$$A = \begin{bmatrix} -4 & 2 \\ -4 & 5 \end{bmatrix}.$$

a) Compute all eigenvalues of A BY HAND.

b) Check that $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ are eigenvectors corresponding to the eigenvalues you found in a) BY HAND.

2) Given the points $(3, 1)$, $(5, -2)$, $(0, 7)$ and $(6, 3)$ in $\mathbb{R}^2$, find the best-fit **LINE** to the points by

a) Finding a system of linear equations that represents a "solution" to the problem,

b) Writing the problem as a matrix equation $A\vec{x} = \vec{b}$,

c) Finding the system $A^t A\vec{x} = A^t\vec{b}$, computing both $A^t A$ and $A^t\vec{b}$,

d) Solving the system in c) and producing the polynomial.

3) Given the simplified link diagram between webpages P_1, P_2, and P_3 described by

- P_1 links to P_3
- P_2 doesn't link to anything
- P_3 links to P_2 and $P1$,

a) Construct the link matrix A.

b) Find the normalized matrix B.

c) Calculate the PageRank matrix C, using $d = .85 = 17/20$.

3) (continued) d) What number is the matrix C guaranteed to have as an eigenvalue?

e) If an associated eigenvector v to the eigenvalue from d) is

$$\begin{bmatrix} 57 \\ 57 \\ 74 \end{bmatrix}$$

find the PageRank of P_1.

4) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$,

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x - y \\ y \\ x \end{bmatrix}.$$

a) Determine a matrix representation A for T.

b) Recall that $\mathrm{Ran}(T)$ is a subspace. Describe $\mathrm{Ran}(T)$ geometrically and justify your answer.

c) Recall that $\ker(T)$ is a subspace. Find a matrix for the orthogonal projection onto $\ker(T)$ and justify your answer.