Name:

Math 227 Exam 3

April 10, 2023

Directions:

1. WRITE YOUR NAME ON THIS TEST!
2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. If you have a question, raise your hand or come up and ask me.

1) Let

$$A = \begin{bmatrix} -20 & 9 \\ 54 & 25 \end{bmatrix}.$$

a) Compute all eigenvalues of A BY HAND.

b) What is the one vector in $\mathbb{R}^2$ that has no possibility of being an eigenvector for A?

c) If $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is an eigenvector for A, find two other eigenvectors for A.

2) Given the points $(2,1)$, $(6,-3)$, $(8,0)$, $(-1,1)$, and $(5,2)$ in $\mathbb{R}^2$, find the best-fit **LINE** to the points by

a) Finding a system of linear equations that represents a "solution" to the problem,

b) Writing the problem as a matrix equation $A\vec{x} = \vec{b}$,

c) Finding the system $A^tA\vec{x} = A^t\vec{b}$, computing both A^tA and $A^t\vec{b}$,

d) Solving the system in c) and producing the polynomial.

3) Given the simplified link diagram between webpages P_1, P_2, P_3 and P_4 described by

- P_1 links to P_3 and P_4
- P_2 links to P_1 and P_3
- P_3 doesn't link to anything
- P_4 links to P_1, P_2, and P_3,

a) Construct the link matrix A.

b) Find the normalized matrix B.

c) Calculate the PageRank matrix C, using $d = .85 = 17/20$.

3) (continued) d) What number is the matrix C guaranteed to have as an eigenvalue?

e) If an associated eigenvector $\vec{v}$ to the eigenvalue from d) is

$$\begin{bmatrix} 25080 \\ 17600 \\ 35739 \\ 22020 \end{bmatrix}$$

find the PageRank of P_4.

4) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$,

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 9x - 3y + 12z \\ 3x - y + 4z \\ -12x + 4y - 16z \end{bmatrix}.$$

a) Determine a matrix representation A for T.

b) Recall that $\ker(T)$ is a subspace. Find a basis for $\ker(T)$.

c) If you did not do so in part b), find an orthonormal basis for $\ker(T)$.