Exam 2 Fall 21

1) a) addition & scaler multiplication

b) sequences of real numbers

c) points, lines, all of $\mathbb{R}^2$ (plane)

d) points, lines, planes, all of $\mathbb{R}^3$

e) $\det(A) \neq 0$

2) a) $\begin{bmatrix} \cos(-\frac{3\pi}{2}) & -\sin(-\frac{3\pi}{2}) & 0 \\ \sin(-\frac{3\pi}{2}) & \cos(-\frac{3\pi}{2}) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 42 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & -42 & 0 \\ 1/3 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -42 & -10 \\ 1/3 & 0 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

3) a) Matrix

b) $T\left(\begin{bmatrix}1\\0\\0\\0\end{bmatrix}\right) = \begin{bmatrix}-3\\1\\0\end{bmatrix}$

$T\left(\begin{bmatrix}0\\1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}2\\0\\0\end{bmatrix}$

$T\left(\begin{bmatrix}0\\0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\1\\0\end{bmatrix}$

$T\left(\begin{bmatrix}0\\0\\0\\1\end{bmatrix}\right) = \begin{bmatrix}0\\0\\0\end{bmatrix}$

$$A = \begin{bmatrix}-3 & 2 & 1 & 0\\ 1 & 0 & 1 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

Check: $A \cdot \begin{bmatrix}x\\y\\z\\w\end{bmatrix} = \begin{bmatrix}z-3x+2y\\ x+z\\ 0\end{bmatrix} = T\left(\begin{bmatrix}x\\y\\z\\w\end{bmatrix}\right)$

– or –

$$T\left(\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ w_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ w_2 \end{bmatrix}\right)$$

$$= T\left(\begin{bmatrix} x_1+x_2 \\ y_1+y_2 \\ z_1+z_2 \\ w_1+w_2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} (z_1+z_2) - 3(x_1+x_2) + 2(y_1+y_2) \\ (x_1+x_2) + (z_1+z_2) \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} z_1 - 3x_1 + 2y_1 + z_2 - 3x_2 + 2y_2 \\ x_1 + z_1 \quad + \quad x_2 + z_2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} z_1 - 3x_1 + 2y_1 \\ x_1 + z_1 \\ 0 \end{bmatrix} + \begin{bmatrix} z_2 - 3x_2 + 2y_2 \\ x_2 + z_2 \\ 0 \end{bmatrix}$$

$$= T\left(\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ w_1 \end{bmatrix}\right) + T\left(\begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ w_2 \end{bmatrix}\right)$$

If $c \in \mathbb{R}$,

$$T\left(c \cdot \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = T\left(\begin{bmatrix} cx \\ cy \\ cz \\ cw \end{bmatrix}\right)$$

$$= \begin{bmatrix} cz - 3cx + 2cy \\ cx + cz \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} c(z - 3x + 2y) \\ c(x + z) \\ 0 \end{bmatrix}$$

$$= c \begin{bmatrix} z - 3x + 2y \\ x + z \\ 0 \end{bmatrix}$$

$$= c \cdot T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \right)$$

4) $\mathcal{F}(\mathbb{R})$ is all functions from $\mathbb{R}$ to $\mathbb{R}$.

So if $f \in \mathcal{F}(\mathbb{R})$,

$f(x) \in \mathbb{R}$ for all $x \in \mathbb{R}$.

In particular,

$f(12) \in \mathbb{R}$, $f(7) \in \mathbb{R}$,

so $f(12) - f(7) \in \mathbb{R}$

Therefore, $W = \mathcal{F}(\mathbb{R})$ and

so is a subspace of $\mathcal{F}(\mathbb{R})$

— or —

1) Zero vector

$0_V(x) = 0$ for all x

$0_V(6) = 0$

$0_V(22) = 0$

$0 - 0 = 0 \in \mathbb{R}$

2) Suppose $f(6) - f(22) \in \mathbb{R}$

and $g(6) - g(22) \in \mathbb{R}$.

Then

$(f+g)(6) - (f+g)(22)$

$= f(6) + g(6) - (f(22) + g(22))$

$= f(6) + g(6) - f(22) - g(22)$

$= (f(6) - f(22)) + (g(6) - g(22))$

$f(6)-f(22) \in \mathbb{R}$

and $g(6)-g(22) \in \mathbb{R}$,

so $(f(6)-f(22)) + (g(6)-g(22)) \in \mathbb{R}$

3) If $c \in \mathbb{R}$ and $f(6)-f(22) \in \mathbb{R}$,

$(c \cdot f)(6) - (c \cdot f)(22)$

$= c \cdot f(6) - c \cdot f(22)$

$= c \cdot (f(6) - f(22))$

$f(6)-f(22) \in \mathbb{R}$ and $c \in \mathbb{R}$, so

$c \cdot (f(6) - f(22)) \in \mathbb{R}$

5) If $(x-z)^4 = (y-z)^4$, then

taking 4^{th} roots,

$$|x-z| = |y-z|$$

So either

$x - z = y - z$, in

which case $x = y$, or

$x - z = -(y-z)$ in

which case

$$2z = x+y$$

$x = y$

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \in S \qquad \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \in S$$

$(x_1 - z_1)^2 = (-1)^2 = 1$ $\qquad$ $(x_2 - z_2)^2 = (-1)^2 = 1$

$(y_1 - z_1)^2 = (-1)^2 = 1$ $\qquad$ $(y_2 - z_2)^2 = 1^2 = 1$

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix} \qquad \begin{aligned} (2-4)^2 &= (-2)^2 = 4 \\ (4-4)^2 &= 0^2 = 0 \end{aligned}$$

$4 \neq 0$, so the

sum is not in S.

(full credit just
for finding
the vectors)