Exam 3 Fall 23

1) a) $0 = \det(A - \lambda I_2)$

$= \det\left(\begin{bmatrix} -68-\lambda & 46 \\ -138 & 93-\lambda \end{bmatrix}\right)$

$= \lambda^2 - 25\lambda + 6348 - 6324$

$= \lambda^2 - 25\lambda + 24$

$= (\lambda - 24)(\lambda - 1)$

$\lambda = 1, 24$

b) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

c) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ since $\begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 \end{bmatrix} = 0$

2) a) $y = mx + b$

$4 = 2m + b$

$3 = -3m + b$

$0 = 4m + b$

$2 = m + b$

b) $$\begin{bmatrix} 1 & 2 \\ 1 & -3 \\ 1 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 0 \\ 2 \end{bmatrix}$$

c) $$A^t = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & -3 & 4 & 1 \end{bmatrix}$$

$$A^t \cdot \vec{b} = \begin{bmatrix} 9 \\ 1 \end{bmatrix}$$

$$A^t \cdot A = \begin{bmatrix} 4 & 4 \\ 4 & 30 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 4 \\ 4 & 30 \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix} = \begin{bmatrix} 9 \\ 1 \end{bmatrix}$$

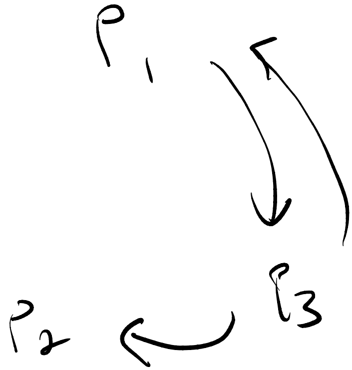
d) $\det\left(\begin{bmatrix} 4 & 4 \\ 4 & 30 \end{bmatrix} \right) = 120 - 16 = 104 \neq 0$

$$\begin{bmatrix} b \\ m \end{bmatrix} = \frac{1}{104} \begin{bmatrix} 30 & -4 \\ -4 & 4 \end{bmatrix} \begin{bmatrix} 9 \\ 1 \end{bmatrix}$$

$$= \frac{1}{104} \begin{bmatrix} 266 \\ -32 \end{bmatrix}$$

$$y = \frac{-32}{104} x + \frac{266}{104}$$

3)



a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

b) $A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$B = \begin{bmatrix} 0 & 1/3 & 1/2 \\ 0 & 1/3 & 1/2 \\ 1 & 1/3 & 0 \end{bmatrix}$

c) $C = \frac{17}{20} B + \frac{1 - \frac{17}{20}}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} \frac{1}{20} & 1/3 & 19/40 \\ \frac{1}{20} & 1/3 & 19/40 \\ \frac{9}{10} & 1/3 & 1/20 \end{bmatrix}$$

d) $\lambda = 1$

e) $\begin{bmatrix} 57 \\ 57 \\ 74 \end{bmatrix}$

4) a) $T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right)=\begin{bmatrix}1\\-7\end{bmatrix}$

$T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right)=\begin{bmatrix}6\\-42\end{bmatrix}$

$T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right)=\begin{bmatrix}-3\\21\end{bmatrix}$

$$A=\begin{bmatrix}1 & 6 & -3\\-7 & -42 & 21\end{bmatrix}$$

b) $\begin{bmatrix}0\\1\\2\end{bmatrix}, \begin{bmatrix}3\\0\\1\end{bmatrix}$

c) Since T is not the zero function and ker(T) has two non parallel vectors in it, ker(T) is a plane

d) $\text{Ran}(T) = \text{col}(A) =$ multiples of $\begin{bmatrix} 1 \\ -7 \end{bmatrix}$

Normalize: $\vec{u} = \frac{1}{\sqrt{50}} \begin{bmatrix} 1 \\ -7 \end{bmatrix}$

Closest vector to $\begin{bmatrix} 7 \\ 2 \end{bmatrix}$ is

$$\vec{u} \cdot \vec{u}^{t} \cdot \begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

$$= \frac{1}{50} \begin{bmatrix} 1 \\ -7 \end{bmatrix} \begin{bmatrix} 1 & -7 \end{bmatrix} \begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

$$= \frac{-7}{50} \begin{bmatrix} 1 \\ -7 \end{bmatrix}$$