Name:

Math 227 Final

April 22, 2013

Directions: WRITE YOUR NAME ON THIS TEST! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. DO NOT convert irrational numbers such as $\sqrt{3}$ or π into decimal approximations; just leave them as they are.

Now, if you had a choice on whether to take this exam, you must also sign to indicate your understanding of the following statement:

I understand that in choosing to take this final, I may lower my grade from what it was before the final. I accept the consequences of my choice to do so.

SIGNED:__________________________________

1) (2 points each) True/False. No justification is necessary.

a) Every symmetric $n \times n$ matrix is diagonalizable.

b) A system of 4 linear equations in 5 unknowns always has infinitely many solutions.

c) For all A, B in $M_n(\mathbb{R})$, it is always true that $AB = BA$.

d) If x is a least-squares solution to $Ax = b$ then $A^t Ax = A^t b$.

e) Every linear map in between finite-dimensional vector spaces may be written as a matrix.

2) (2 points each blank) Fill in the blank.

a) The only possible eigenvalues of an orthogonal matrix are ____________.

b) An invertible matrix cannot have determinant equal to ____________.

c) A basis for a vector space V is both linearly __________________ and __________________.

d) If A is an $m \times n$ matrix, then A^t is an ____________ matrix.

3) Propane (C_3H_8) combines with oxygen (O) to produce water (H_2O) and carbon dioxide (CO_2) via the equation

$$C_3H_8 + O_2 \rightarrow H_2O + CO_2.$$

a) (8 points) Determine a system of linear equations (or a matrix) that balances the equation.

b) (8 points) Balance the equation. *Note:* If you can do this without using part a), you will get full credit for the entire problem.

4) Let $v = \langle 5, 5\sqrt{3}, 0\rangle$ and $w = \langle 5, 5\sqrt{3}, 10\rangle$.

a) (6 points) Calculate $\|v\|_2$, $\|w\|_2$, and $v \cdot w$.

b) (8 points) Give a formula for the angle between the vectors v and w, then find the angle between them, correct to the nearest degree.

5) Find the matrix of the linear transformations on $\mathbb{R}^3$ that, in homogeneous coordinates

a) (4 points) scales a 2-vector up by a factor of 5

b) (5 points) shifts a 2-vector down 3 units and right 9 units

c) (6 points) rotates a 2-vector by $\pi/3$ radians counterclockwise

d) (7 points) first scales a vector up by a factor of 5, then shifts the vector down 7 units and right 9 units, then rotates the vector by $\pi/3$ radians counterclockwise.

6) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$,

$$T(x, y) = (y, x - y, 0).$$

a) (8 points) Show that T is linear.

b) (5 points) Find $Null(T)$.

c) (5 points) Find the rank of T.

7) (18 points) Given the simplified link diagram between webpages P_1, P_2, P_3 and P_4 described by

- P_1 links to P_3
- P_2 links to P_1 and P_4
- P_3 links to P_1, P_2 and P_4
- P_4 doesn't link to anything

find the PageRank of P_1, using $d = .85$. SHOW YOUR STEPS.

8) Let $A = \begin{bmatrix} 33/10 & 19/10 \\ -37/10 & 9/10 \end{bmatrix}$.

a) (7 points) Calculate the norm of A. Show the procedure of arriving at your answer.

b) (12 points) Determine the polar decomposition of A.

9) Given the signals $x_k = k$ and $z_k = 2^k$ and the homogeneous linear difference equation

$$-y_{k+3} + 4y_{k+2} - 5y_{k+1} + 2y_k = 0, \tag{1}$$

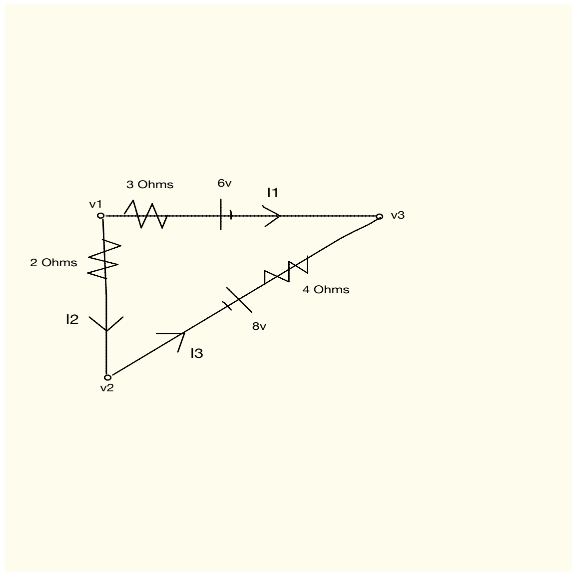
a) (10 points) check that (x_k) and (z_k) all satisfy equation (1);

b) (5 points) determine the Casorati matrix associated to the signals (x_k) and (z_k).

c) (5 points) Is $\{(x_k), (z_k)\}$ a basis for the subspace of all signals satisfying equation (1)? Justify your assertion.

10) (15 points) Given the points $(0, 1)$, $(1, 5)$, and $(3, 8)$ in $\mathbb{R}^2$, find the best-fit line through these three points.

11) Consider the following electrical circuit:



a) (6 points) Find the edge-node incidence matrix A.

b) (4 points) Determine the resistance matrix R.

c) (10 points) Find the currents I_1, I_2 and I_3 and potential differences between v_1, v_2 and v_3.

12) Let W be the collection of all vectors w in $\mathbb{R}^3$ whose second coordinate is even.

a) (8 points) Show that W is NOT a subspace of $\mathbb{R}^3$.

b) (10 points) If v is a vector in $\mathbb{R}^3$ and $v \cdot w = 0$ for all $w \in W$, show that v must be the zero vector.

BONUS 1: (10 points) For all counting numbers n and for all orthogonal projections P in $M_n(\mathbb{R})$, show that if $P \neq I_n$, then $\det(P) = 0$.

BONUS 2: (10 points) Let $\mathbb{P}$ be the real vector space of all polynomials with real coefficients. Define $T : \mathbb{P} \to \mathbb{P}$ by

$$T(p(x)) = p(x^2)$$

for all polynomials p. Show that T is linear.

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1 2 3 4 5 WeBWorK assignments made me do more homework than I would have done without the assignments.

1 2 3 4 5 WeBWorK helped improve my course grade by preparing me for exams.

1 2 3 4 5 WeBWorK helped improve my course grade because of the extra practice.

1 2 3 4 5 WeBWorK caused me to interact with my fellow students, or professor, or tutors because I discussed problems with them.

1 2 3 4 5 I wish I had WeBWorK assignments in other mathematics classes.

Circle one: WeBWorK questions were too easy just right too difficult

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