

Math 227 Final Winter 13 Solutions

1) (2 points each)

2)

3)

$$\text{a) } x_1 \begin{bmatrix} 3 \\ 8 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ -2 \\ -1 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

b) $x_1 = 1, x_2 = 5, x_3 = 4, x_4 = 3$ is one possible solution.

4) a) $\|v\|_2 = 10, \|w\|_2 = 10\sqrt{2}, v \cdot w = 100.$

$$\text{b) } \cos(\theta) = \frac{v \cdot w}{\|v\|_2 \cdot \|w\|_2}, \theta = 45^\circ$$

6) a) Do it.

$$\text{b) } \text{Nul}(T) = \{(0, 0)\}$$

$$\text{c) } \text{Rank}(T) = 2$$

7) $4389/17165 \approx .25569.$

8) a) $\|A\| = 5$

$$\text{b) } |A| = \begin{bmatrix} 247/50 & 21/50 \\ 21/50 & 103/50 \end{bmatrix}, w = \frac{1}{5} \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$$

9) a) Do it.

$$\text{b) } \begin{bmatrix} k & 2^k \\ k+1 & 2^{k+1} \end{bmatrix}$$

c) No- the solution space has dimension 3.

10) $y = 31/14x + 12/7.$

$$\text{11) a) } \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\text{b) } \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\text{c) } I_1 = -14/9, I_2 = I_3 = 14/9, v_1 - v_2 = 4/3, v_2 - v_3 = -16/9, \\ v_1 - v_3 = -4/9$$

12) a) $v = (0, 2, 0)$ is in W , but $\sqrt{2}v$ is not.

b) We have $(x, 2, z) \in W$ for all $x, z \in \mathbb{R}$ and $(x, 0, z) \in W$ implies $v \cdot (x, 0, z) = 0$. Then if $y \neq 0$, $(2x/y, 2, 2z/y) \in W$, and so $v \cdot (2x/y, 2, 2z/y) = 0$. By factoring out $2/y$, we get that $v \cdot (x, y, z) = 0$ for all $x, y \in \mathbb{R}$ and for all $y \neq 0$. This then implies that $v = 0$.