Name:

Math 227 Final

April 20, 2015

Directions: WRITE YOUR NAME ON THIS TEST! Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer. Decimal approximations, accurate to four decimal places, are acceptable.

1) Let $v = \langle 2, 10\sqrt{2}, 2\rangle$ and $w = \langle 0, \sqrt{6}, 3\rangle$. Note that v and w are linearly independent.

a) (3 points) Calculate $\|v\|_\infty$ and $\|v\|_2$.

b) (3 points) Calculate $\|w\|_0$ and $\|w\|_1$.

c) (5 points) Calculate $\langle v, w\rangle = v \cdot w$ and use this to orthogonalize v and w via Gram-Schmidt.

2) Potassium Arsenate (K_3AsO_4) combines with Hydrogen Disulfide (H_2S) to produce arsenic pentasulfide (As_2S_5), potassium hydroxide (KOH), and water (H_2O) via the equation

$$K_3AsO_4 + H_2S \to As_2S_5 + KOH + H_2O$$

a) (8 points) Determine a system of linear equations (or a matrix) that balances the equation.

b) (8 points) Balance the equation. *Note:* If you can do this without using part a), you will get full credit for the entire problem.

3) Find the matrix of the linear transformations on $\mathbb{R}^3$ that, in homogeneous coordinates,

a) (3 points) scales the x-coordinate of a 2-vector down by 3 and the y-coordinate up by 7.

b) (4 points) shifts a 2-vector up 13 units and left 8 units

c) (5 points) rotates a 2-vector by $2\pi/3$ radians counterclockwise

d) (6 points) scales the x-coordinate of a 2-vector down by 3 and the y-coordinate up by 7, then rotates the vector by $2\pi/3$ radians counterclockwise, and finally shifts the vector up 13 units and left 8 units.

4) Find the interpolating cubic through the points $(0, 6)$, $(-1, 2)$, $(-5, 7)$ and $(3, 4)$ in $\mathbb{R}^2$ by,

a) (6 points) writing down a system of linear equations that determines the coefficients of the polynomial, then

b) (4 points) finding the solution to the equation and writing down the polynomial.

5) Given the points $(0, 6)$, $(-1, 2)$, $(-5, 7)$ and $(3, 4)$ in $\mathbb{R}^2$, find the best-fit quadratic to the points by

a) (6 points) Finding a system of linear equations that represents a "solution" to the problem,

b) (5 points) Writing the problem as a matrix equation $Ax = b$,

c) (5 points) Finding the system $A^tAx = A^tb$, computing both A^tA and A^tb,

d) (6 points) Solving the system in c) and producing the polynomial.

6) Given the simplified link diagram between webpages P_1, P_2, P_3, P_4 and P_5 described by

- P_1 links to P_2 and P_5
- P_2 links to P_1
- P_3 links to P_1, P_2, P_4 and P_5
- P_4 doesn't link to anything
- P_5 links to P_2, P_3, and P_4

a) (5 points) Construct the link matrix A.

b) (5 points) Find the normalized matrix B.

c) (8 points) Calculate the PageRank matrix C, using $d = .85 = 17/20$.

d) (6 points) Find the associated eigenvector v with all positive entries whose 1-norm is equal to one and find the PageRank of P_2.

7) Given the signals $x_k = 1$, $w_k = (-1/2)^k$, and $z_k = k$ and the homogeneous linear difference equation

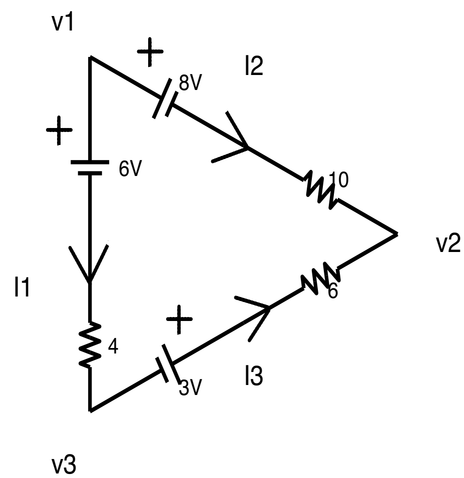
$$2y_{k+3} - 3y_{k+2} + y_k = 0, \tag{1}$$

a) (9 points) check that $(x_k)_{k\in\mathbb{Z}}$, $(w_k)_{k\in\mathbb{Z}}$, and $(z_k)_{k\in\mathbb{Z}}$ all satisfy equation (1);

b) (5 points) determine the Casorati matrix associated to the signals $(x_k)_{k\in\mathbb{Z}}$, $(w_k)_{k\in\mathbb{Z}}$, and $(z_k)_{k\in\mathbb{Z}}$.

c) (6 points) Is $\{(x_k)_{k\in\mathbb{Z}}, (w_k)_{k\in\mathbb{Z}}, (z_k)_{k\in\mathbb{Z}}\}$ a basis for the subspace of all signals satisfying equation (1)? Justify your assertion.

8) Consider the following electrical circuit:



a) (5 points) Find the edge-node incidence matrix A.

b) (3 points) Determine the resistance matrix R.

c) (6 points) Set up a matrix equation for finding the currents I_1, I_2, and I_3 and the potential differences between v_1, v_2, and v_3.

d) (6 points) Find the currents I_1, I_2 and I_3 and potential differences between v_1, v_2 and v_3.

9) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$,

$$T(x, y, z) = (3x - y + z, 10x + 4y - 8z, 2y - 9z).$$

a) (8 points) Show that T is linear.

b) (5 points) Find the standard matrix of T and compute all eigenvalues of T. Is T invertible?

c) (10 points) Let $W = \{S : \mathbb{R}^3 \to \mathbb{R}^3 \mid ST = 0\}$. Show that W is a subspace of $M_3(\mathbb{R})$.

10) a) (6 points) Let

$$v_1 = \begin{bmatrix} 7 \\ -3 \\ 11 \\ 42 \end{bmatrix}, \ v_2 = \begin{bmatrix} 12 \\ 9 \\ -1 \\ -15 \end{bmatrix}$$

Find two different vectors v and w in $span\{v_1, v_2\}$ that are neither a scalar multiple of v_1 nor a scalar multiple of v_2. Then find a vector u that is NOT in $span\{v_1, v_2\}$. For the last part, be sure to show that your answer is correct.

b) (10 points) If $A \in M_n(\mathbb{R})$ is invertible and $B \in M_n(\mathbb{R})$ is not invertible, show that AB is not invertible.