Name:

Math 227 Final

Winter 2020 Corona Virus Edition

Directions:

1. You may use your notes, textbook, and a calculator for this exam, but NO OTHER RESOURCES; if I can determine you're cheating on this exam, you'll get a zero.
2. Except where indicated, merely finding the answer to a problem is not enough to receive full credit; you must show how you arrived at that answer.
3. Unless otherwise indicated, decimal approximations for a numerical answer accurate to 4 decimal places are acceptable.
4. Once you submit this exam through Canvas, it will factor into your grade in the manner described in the syllabus. NO TAKE-BACKS. Please indicate your understanding of the potential consequences of taking this exam by signing the statement below:

I understand that by taking this exam, I may lower my grade from what it was before the final.

Signed: ____________________________

1) Given the simplified link diagram between webpages P_1, P_2, and P_3 described by

- P_1 links to P_3
- P_2 links to P_1 and P_3
- P_3 doesn't link to anything,

a) Construct the link matrix A.

b) Find the normalized matrix B.

c) Calculate the PageRank matrix C, using $d = .85 = 17/20$.

d) Find all eigenvalues of C BY HAND (recall that $\lambda = 1$ should always be an eigenvalue).

e) If $v = \begin{bmatrix} 20/37 \\ 800/2109 \\ 1 \end{bmatrix}$ is an eigenvector for $\lambda = 1$, find the PageRank of P_2.

2) Given the points $(-1, 2)$, $(0, 8)$, and $(3, -1)$ in $\mathbb{R}^2$, find the best-fit line to the points by

a) Finding a system of linear equations that represents a "solution" to the problem,

b) Writing the problem as a matrix equation $Ax = b$,

c) Finding the system $A^tAx = A^tb$, computing both A^tA and A^tb,

d) Solving the system in c) BY HAND and producing the polynomial.

3) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} y - x \\ y \\ 3x + y \end{bmatrix}.$$

a) Determine a matrix representation A for T.

b) Find a basis for the column space of A, with justification as to why your answer is a basis.

c) Find an orthonormal basis for the column space of A (an answer for this will count as an answer for a)), showing all work used to constuct this basis.

4) Let $\mathcal{S}$ denote the vector space of all sequences of real numbers. Let W be the subset of $\mathcal{S}$ consisting of all convergent sequences. Show that W is a subspace of $\mathcal{S}$.

5) We know from work in class that the kernel of a linear transformation is a subspace. For ALL subspaces W of $\mathbb{R}^n$, show that there is a linear map $T : \mathbb{R}^n \to \mathbb{R}^n$ with $\ker(T) = W$. *Hint:* orthogonal projections.

6) Let $N \in M_n(\mathbb{R})$ be a nilpotent matrix. The *nilpotency index* of N is the smallest whole number k with $N^k = 0$ (since N is nilpotent, we know such a k exists). Do your best to show that for ALL values of n and ALL nilpotent matrices $N \in M_n(\mathbb{R})$, the nilpotency index of N can never be greater than n.