Winter 18 Exam 2

1) a) $\begin{bmatrix} 17 & 0 & 0 \\ 0 & 17 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b $\begin{bmatrix} 1 & 0 & 11 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} \cos(-\pi/6) & -\sin(-\pi/6) & 0 \\ \sin(-\pi/6) & \cos(-\pi/6) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

d)

a) · c) · b)

$$\begin{bmatrix} \frac{17\sqrt{3}}{2} & \frac{17}{2} & -34 + \frac{187\sqrt{3}}{2} \\ -\frac{17}{2} & \frac{17\sqrt{3}}{2} & -\frac{187}{2} - 34\sqrt{3} \\ 0 & 0 & 1 \end{bmatrix}$$

2) a)

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 1 \\ 2 & 1 \\ 1 & 4 \end{bmatrix}$$

b) $$A^t A = \begin{bmatrix} 6 & 5 \\ 5 & 18 \end{bmatrix}$$

$$\det(A^t A) = 83 > 0$$

so $A^t A$ is invertible

c) $P = A(A^tA)^{-1}A^t$

$$= \frac{1}{83}\begin{bmatrix} 34 & -35 & 21 \\ -35 & 58 & 15 \\ 21 & 15 & 74 \end{bmatrix}$$

3) a)

$$b = 6$$

$$-m + b = 2$$

$$-5m + b = 7$$

$$3m + b = 4$$

b)

$$\begin{bmatrix} 0 & 1 \\ -1 & 1 \\ -5 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 7 \\ 4 \end{bmatrix}$$

c)

$$\begin{bmatrix} 35 & -3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} -25 \\ 19 \end{bmatrix}$$

d) $$\begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 35 & -3 \\ -3 & 4 \end{bmatrix}^{-1} \begin{bmatrix} -25 \\ 19 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{43}{131} \\ \frac{590}{131} \end{bmatrix}$$

$$y = -\frac{43}{131} x + \frac{590}{131}$$

4) a) i) zero vector

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad 0+0=0 \checkmark$$

ii) scalar multiples: let

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in W, \quad \text{so } a+d=0$$

k a scalar,

$$k\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$$

$$ka+kd = k(a+d)$$
$$= k \cdot 0 = 0 \checkmark$$

iii) Addition

Let $\begin{bmatrix} a & b \\ c & d \end{bmatrix}, \begin{bmatrix} x & y \\ z & w \end{bmatrix} \in W$

so $a+d = b+w = 0$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} a+x & b+y \\ c+z & d+w \end{bmatrix}$$

$$(a+x)+(d+w)$$

$$= (a+d)+(x+w)$$

$$= 0+0 = 0 \quad \checkmark$$

so W is a s-bspace.

b) Let $v = (0,1) \in S$

and let $c = -1$.

Then $cv = (0,-1) \notin S$

since $-1 < 0$, so S is not a subspace since S is not closed under scalar multiplication.

Bonus: Suppose

$$T(p(x)) = 0.$$

Then $p'(x) = 0$. But the only polynomials with derivatie equal to zero are constant polynomials, and so

$$\ker(T) = \{\text{constant polynomials}\}$$