1) a) $\det\left(\begin{bmatrix} 3 & -4 \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)$

$= \det\left(\begin{bmatrix} 3-\lambda & -4 \\ 1 & -2-\lambda \end{bmatrix}\right)$

$= \lambda^2 - \lambda - 2$

$= (\lambda - 2)(\lambda + 1)$

$0 = (\lambda - 2)(\lambda + 1)$

$\lambda = -1, 2$

b) eigenvectors

$$\begin{bmatrix} 4 \\ 1 \end{bmatrix} \quad (\lambda = 2)$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (\lambda = -1)$$

c) $$\begin{bmatrix} 3 & -4 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \end{bmatrix}$$

$$= 2 \begin{bmatrix} 4 \\ 1 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} 3 & -4 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$= (-1) \begin{bmatrix} 1 \\ 1 \end{bmatrix} \checkmark$$

2)

$$\begin{tikzcd} P_1 \arrow[rr] \arrow[d] & & P_2 \\ P_3 \arrow[urr] & & \end{tikzcd}$$

a) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = A$

b) $A \mapsto \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$$B = \begin{bmatrix} 0 & 1/3 & 0 \\ 1/2 & 1/3 & 1 \\ 1/2 & 1/3 & 0 \end{bmatrix}$$

c) $C = \frac{17}{20}\begin{bmatrix} 0 & 1/3 & 0 \\ 1/2 & 1/3 & 1 \\ 1/2 & 1/3 & 0 \end{bmatrix}$

$+ \frac{1}{20}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1/20 & 1/3 & 1/20 \\ 19/40 & 1/3 & 9/10 \\ 19/40 & 1/3 & 1/20 \end{bmatrix}$

d)

$$\begin{bmatrix} 40/57 \\ 37/20 \\ 1 \end{bmatrix}$$

1-norm $\frac{40}{57} + \frac{37}{20} + 1$

$$= \frac{800 + 2109 + 1140}{1140}$$

$$= \frac{4049}{1140}$$

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$$\frac{40}{57} \cdot \frac{1140}{4049} = \frac{800}{4049}$$

$$\approx .1976$$

3) a)

$$T\begin{bmatrix}1\\0\\0\\0\end{bmatrix}=\begin{bmatrix}9\\-2\end{bmatrix}$$

$$T\begin{bmatrix}0\\1\\0\\0\end{bmatrix}=\begin{bmatrix}-2\\-1\end{bmatrix}$$

$$T\begin{bmatrix}0\\0\\1\\0\end{bmatrix}=\begin{bmatrix}0\\-12\end{bmatrix}$$

$$T\begin{bmatrix}0\\0\\0\\1\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$$

$$A=\begin{bmatrix}9 & -2 & 0 & 1\\-2 & -1 & -12 & 0\end{bmatrix}$$

b) $\operatorname{rref}\begin{bmatrix} 9 & -2 & 0 & 1 & 0 \\ -2 & -1 & -12 & 0 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 24/13 & 1/13 & 0 \\ 0 & 1 & \frac{108}{13} & \frac{-2}{13} & 0 \end{bmatrix}$$

$$x = -\frac{24}{13} z - \frac{w}{13}$$

$$y = \frac{-108}{13} z + \frac{2w}{13}$$

$z = 13, \ w = 0$ $\quad \begin{bmatrix} -24 \\ -108 \\ 13 \\ 0 \end{bmatrix}$

$w = 13, \ z = 0$ $\quad \begin{bmatrix} -1 \\ 2 \\ 0 \\ 13 \end{bmatrix}$

$$\left\{ \begin{bmatrix} -24 \\ -108 \\ 13 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \\ 13 \end{bmatrix} \right\}$$

lin ind since not scalar

multiples, dimension 2

c) $\frac{1}{\sqrt{12409}} \begin{bmatrix} -24 \\ -108 \\ 13 \\ 0 \end{bmatrix}$,

$$\begin{bmatrix} -187\sqrt{\frac{7}{22261746}} \\ 157\sqrt{\frac{2}{7791611}} \\ 32\sqrt{\frac{6}{25972037}} \\ \sqrt{12409/12558} \end{bmatrix}$$

4) a) $T(cx)$

$= cT(x)$

$= c\lambda x$

$= \lambda(cx)$

b) Suppose

$$a(v_1+v_2)+b(v_2+v_3)+c(v_3+v_1)=0$$

Then

$$(a+c)v_1+(a+b)v_2+(b+c)v_3=0$$

Since $\{v_1, v_2, v_3\}$ is a basis,

$$a+c=a+b=b+c=0$$

so $a=-c$

$a=-b$

$b=-c$

But then $a=-c=b=-b$

so $b=0$ and so $a=c=0$.

Since the dimension is 3, we have a basis.