

Final '18

$$\begin{bmatrix} 4 & 1 & -3 & 11 \\ 2 & 3 & -1 & 1 \\ 3 & -2 & 5 & 21 \end{bmatrix}$$

$-2R_2 + R_1$

$$\begin{bmatrix} 0 & -5 & -1 & 9 \\ 2 & 3 & -1 & 1 \\ 3 & -2 & 5 & 21 \end{bmatrix}$$

$-3/2 R_2 + R_3$

$$\begin{bmatrix} 0 & -5 & -1 & 9 \\ 2 & 3 & -1 & 1 \\ 0 & -\frac{13}{2} & \frac{13}{2} & \frac{39}{2} \end{bmatrix}$$

$$\begin{bmatrix} 0 & -5 & -1 & 9 \\ 2 & 3 & -1 & 1 \\ 0 & -\frac{13}{2} & \frac{13}{2} & \frac{39}{2} \end{bmatrix}$$

$2R_3$

$$\begin{bmatrix} 0 & -5 & -1 & 9 \\ 2 & 3 & -1 & 1 \\ 0 & -13 & 13 & 39 \end{bmatrix}$$

$R_3 + R_2$

$$\begin{bmatrix} 0 & -5 & -1 & 9 \\ 2 & -10 & 12 & 40 \\ 0 & -13 & 13 & 39 \end{bmatrix}$$

$-2R_1 + R_2$

$$\begin{bmatrix} 0 & -5 & -1 & 9 \\ 2 & 0 & 14 & 22 \\ 0 & -13 & 13 & 39 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -5 & -1 & 9 \\ 2 & 0 & 14 & 22 \\ 0 & -13 & 13 & 39 \end{bmatrix}$$

$$3R1 + R3$$

$$\begin{bmatrix} 0 & -5 & -1 & 9 \\ 2 & 0 & 14 & 22 \\ 0 & -78 & 0 & 156 \end{bmatrix}$$

$$R3 / -78, R2 / 2$$

$$\begin{bmatrix} 0 & -5 & -1 & 9 \\ 1 & 0 & 7 & 11 \\ 0 & 1 & 0 & -2 \end{bmatrix}$$

$$5R3 + R1$$

$$\begin{bmatrix} 0 & 0 & -1 & -1 \\ 1 & 0 & 7 & 11 \\ 0 & 1 & 0 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & -1 & -1 \\ 1 & 0 & 7 & 11 \\ 0 & 1 & 0 & -2 \end{bmatrix}$$

$$7R1 + 2R2$$

$$\begin{bmatrix} 0 & 0 & -1 & -1 \\ 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -2 \end{bmatrix}$$

$$z=1, \quad x=4, \quad y=-2$$

$$2) \det \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 6 \\ 0 & 2 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right)$$

$$= \det \begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & -\lambda & 6 \\ 0 & 2 & -1-\lambda \end{bmatrix}$$

$$= \begin{array}{cccccc} 1-\lambda & 0 & 0 & 1-\lambda & 0 \\ 0 & -\lambda & 6 & 0 & -\lambda \\ 0 & 2 & -1-\lambda & 0 & 2 \end{array}$$

+1

$$= (1-\lambda)(\lambda)(1+\lambda) - 12(1-\lambda)$$

$$= (1-\lambda)(\lambda^2 + \lambda - 12)$$

$$= (1-\lambda)(\lambda+4)(\lambda-3) = 0$$

$$\lambda = 1, -4, 3$$

$$b) \begin{bmatrix} 0 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\lambda = -4$ $\lambda = 3$ ($\lambda = 1$)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 6 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ -8 \end{bmatrix} = -4 \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 6 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 3 \end{bmatrix} = 3 \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 6 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$3 \text{ a) } \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{b) } \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{c) } \begin{bmatrix} \cos(3\pi/4) & -\sin(3\pi/4) & 0 \\ \sin(3\pi/4) & \cos(3\pi/4) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

d) b' r i c

$$= \begin{bmatrix} -\frac{1}{4\sqrt{2}} & -\frac{1}{4\sqrt{2}} & 7 \\ \frac{1}{4\sqrt{2}} & -\frac{1}{4\sqrt{2}} & -\frac{1}{b} \\ b & b & 1 \end{bmatrix}$$

$$4) a) \quad 4 = a + b + c + d$$

$$6 = -27a + 9b - 3c + d$$

$$2 = 8a + 4b + 2c + d$$

$$7 = -a + b - c + d$$

$$b) \quad \text{ref} \begin{bmatrix} 1 & 1 & 1 & 1 & 4 \\ -27 & 9 & -3 & 1 & 6 \\ 8 & 4 & 2 & 1 & 2 \\ -1 & 1 & -1 & 1 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 1/15 \\ 0 & 1 & 0 & 0 & -3/10 \\ 0 & 0 & 1 & 0 & -47/30 \\ 0 & 0 & 0 & 1 & 29/5 \end{bmatrix}$$

$$y = \frac{x^3}{15} - \frac{3x^2}{10} - \frac{47x}{30} + \frac{29}{5}$$

$$5) a) 4 = a + b + c$$

$$6 = 9a - 3b + c$$

$$2 = 4a + 2b + c$$

$$7 = a - b + c$$

$$b) \begin{bmatrix} 1 & 1 & 1 \\ 9 & -3 & 1 \\ 4 & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 2 \\ 7 \end{bmatrix}$$

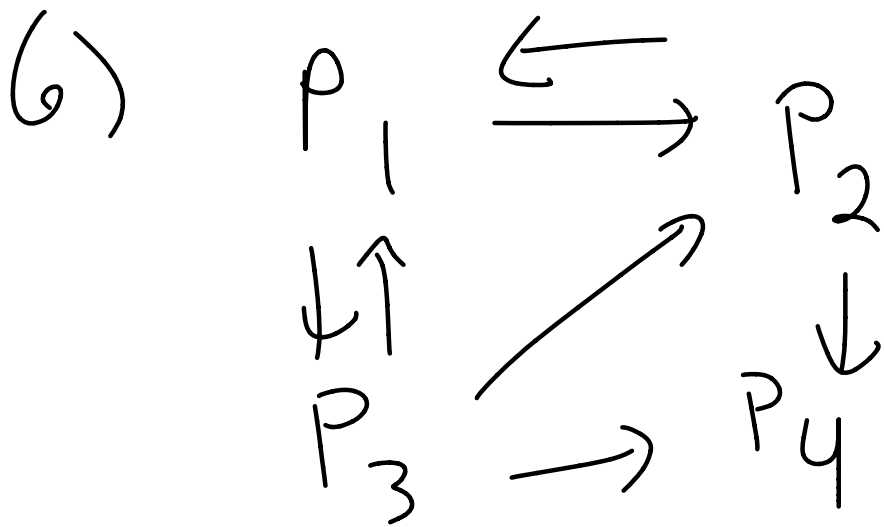
$$c) \begin{bmatrix} 99 & -19 & 15 \\ -19 & 15 & -1 \\ 15 & -1 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 73 \\ -17 \\ 19 \end{bmatrix}$$

d)

$$\text{rref} \begin{bmatrix} A^t & A & b \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & -\frac{161}{398} \\ 0 & 1 & 0 & -\frac{497}{398} \\ 0 & 0 & 1 & \frac{1185}{199} \end{bmatrix}$$

$$y = \frac{-161}{398} x^2 - \frac{497}{398} x + \frac{1185}{199}$$



a)

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

b) $A \rightarrow$

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & y_2 & y_3 & y_4 \\ y_2 & 0 & y_3 & y_4 \\ y_3 & 0 & 0 & y_4 \\ 0 & y_2 & y_3 & y_4 \end{bmatrix}$$

$$c) C = \frac{17}{20} B + \frac{1 - \frac{17}{20}}{4} E$$

$$= \begin{bmatrix} 0 & 17/40 & 17/40 & 17/80 \\ 17/40 & 0 & 17/40 & 17/80 \\ 17/40 & 0 & 0 & 17/80 \\ 0 & 17/40 & 17/40 & 17/80 \end{bmatrix}$$

$$+ \frac{3}{80} E$$

$$= \begin{bmatrix} 3/80 & 37/80 & \frac{77}{240} & 1/4 \\ 37/80 & 3/80 & \frac{77}{240} & 1/4 \\ 37/80 & 3/80 & 3/80 & 1/4 \\ 3/80 & 37/80 & \frac{77}{240} & 1/4 \end{bmatrix}$$

$$d) \quad v = \begin{bmatrix} 1 \\ 1 \\ 60/77 \\ 1 \end{bmatrix}$$

$$\|v\|_1 = 3 + \frac{60}{77}$$

$$= \frac{291}{77}$$

$$\text{Page Rank of } P3 = \frac{60}{291} \approx 0.2062$$

$$7) \approx \begin{bmatrix} 1542 & 60 \\ 60 & 1416 \end{bmatrix}$$

$$b) A^t A = U D U^t$$

with

$$D = \begin{bmatrix} 1392 & 0 \\ 0 & 1566 \end{bmatrix}$$

$$U = \frac{1}{\sqrt{29}} \begin{bmatrix} -2 & 5 \\ 5 & 2 \end{bmatrix}$$

$$\lambda = 1392, 1566$$

$$c) M = U D V^T$$

with $D = \begin{bmatrix} 3\sqrt{174} & 0 \\ 0 & 4\sqrt{87} \\ 0 & 0 \end{bmatrix}$

$$U = \begin{bmatrix} \sqrt{2/3} & -1/\sqrt{3} & 0 \\ -1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{bmatrix}$$

$$V = \begin{bmatrix} \frac{5}{\sqrt{29}} & \frac{2}{\sqrt{29}} \\ -\frac{2}{\sqrt{29}} & \frac{5}{\sqrt{29}} \end{bmatrix}$$

$$\sigma = 3\sqrt{174}, \quad 4\sqrt{87}$$

≈ 39.5727 ≈ 37.3095

$$\vec{T} = U D$$

$$= \begin{bmatrix} 6\sqrt{2} & -4\sqrt{2} \\ -3\sqrt{2} & -4\sqrt{2} \\ -3\sqrt{2} & -4\sqrt{2} \end{bmatrix}$$

$$1^{\text{st}} \text{ comp: } 3\sqrt{2} \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$$

$$2 \begin{bmatrix} 32.311 \\ -16.1555 \\ -16.1555 \end{bmatrix}$$

8) a)

$$A = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

b)

$R =$

$$\begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 15 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$$d) \text{ret} \left(\begin{bmatrix} R & A & b_1 \\ A^t & 0 & b_4 \\ & & 0 \\ & & 0 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 0 & \text{---} & 0 \\ 0 & 1 & 0 & \text{---} & 0 \\ 0 & 0 & 1 & 0 & \text{---} \\ 0 & 0 & 0 & 1 & 0 & \text{---} & 0 \\ & & & & 1 & 0 & 0 & 1 & 20 \\ 0 & 0 & \text{---} & & 1 & 0 & -1 & -5 \\ 0 & 0 & \text{---} & 0 & & 0 & 1 & -1 & -15 \end{bmatrix}$$

$$I_1 = I_2 = I_3 = I_u = 0 \text{ amps}$$

$$v_1 - v_4 = 20 \text{ V}$$

$$v_2 - v_4 = -5 \text{ V}$$

$$v_3 - v_4 = -5 \text{ V}$$

9)

$$v_1 + v_2 = \begin{bmatrix} 14 \\ 29 \\ -11 \end{bmatrix}$$

$$v_1 - v_2 = \begin{bmatrix} -2 \\ 55 \\ -9 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 29 \\ -11 \end{bmatrix}$$

$$\text{RREF} \begin{bmatrix} 14 & -2 & 0 \\ 29 & 55 & 29 \\ -11 & -9 & -11 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 6 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(b) \quad \tau \begin{bmatrix} - \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 13 \end{bmatrix}$$

$$\tau \begin{bmatrix} 0 \\ - \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ -5 \end{bmatrix}$$

$$\tau \begin{bmatrix} 0 \\ 0 \\ - \\ 0 \end{bmatrix} = \begin{bmatrix} -10 \\ 4 \\ 9 \end{bmatrix}$$

$$\tau \begin{bmatrix} 0 \\ 0 \\ 0 \\ - \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \\ -1 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & 8 & -10 & 6 \\ 6 & 0 & 4 & 9 \\ 13 & -5 & 9 & -1 \end{bmatrix}$$

b) Column space = range

But all of \mathbb{R}^3 is in range,

So $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$11) a) A^t O A = O \quad \checkmark$$

b) let $B \in W, c \in \mathbb{R}$.

$$A^t (cB) A$$

$$= c (A^t B A)$$

$$= cB \quad \checkmark$$

c) let $B_1, B_2 \in W$

$$A^t (B_1 + B_2) A$$

$$= A^t B_1 A + A^t B_2 A$$

$$= B_1 + B_2 \quad \checkmark$$

12) Since this is just \mathbb{R}^2 ,

$$\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$$